4

188 (188)

: HAND WRITTEN NOTES :-

OF

(1)

Electrical ENGG

-: SUBJECT :-

MEASUREMENT

& INSTRUMENTATION

2

Electrical & Electronics Measurement And Instrumentation.

(only for IES/PSU)

3

Electrical	Electronic	Instrumentation.
Measurement of R, L, C	Qmeter	Measurement of non electrical quantities like
	Error Analysis	temp
Measurement of V, I, P, p.f, Energy.	CRO	pressure
	Digital Meter	Humidity
potentiometer		flow. & Acceleration

Instrumentation
Xmer

Text Book :- AK SHAWNEY

Weightage

IES → Conv, Objective
Conv 40, Objective 40 — 80 Marks

GATE → 3-4 Question's (5-6 Marks)

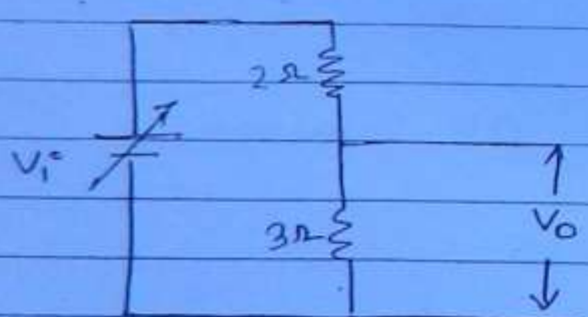
4

Analog Meter

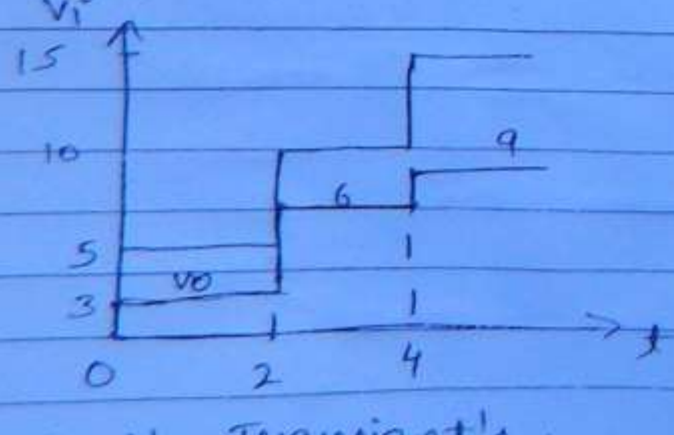
- Quantity of Measurement
  - I
  - V
  - P
  - P.f
  - Energy
  - f.
- principle
  - Electromagnetic
  - Electrostatic
  - Induction
  - Thermal (Heating)
  - :
- Representation
  - Indicating
    - Ammeter
    - Voltmeter
    - Wattmeter
    - pf meter
  - Integrating
    - Energy meter
  - Rec
    - pot
    - Gro

Order of Instruments :-

Zeroeth order :- system change instantenously.



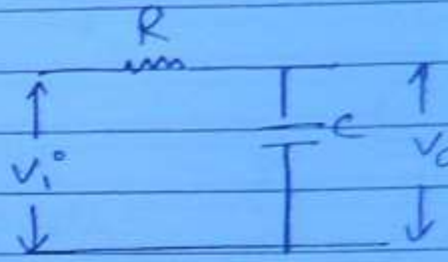
$$V_0 = \frac{3V_i}{3+2} = \frac{3V_i}{5}$$



→ No Transient's.

First order :—

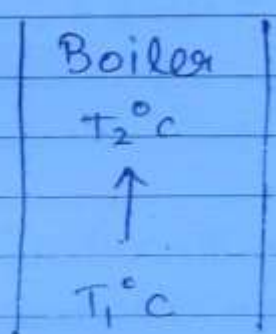
5

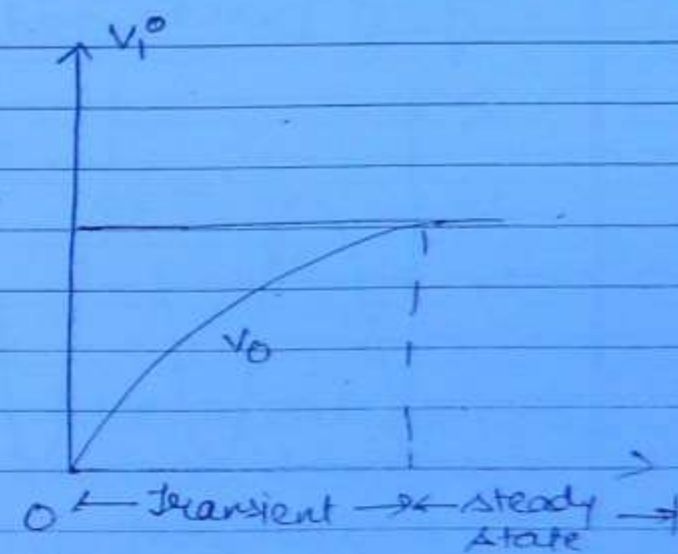


Recording
↓
potentiometric
Galvanometric

Null Detector
↓
potentiometer.

$$V_0 = V_i\left[1 - e^{-t/RC}\right]$$

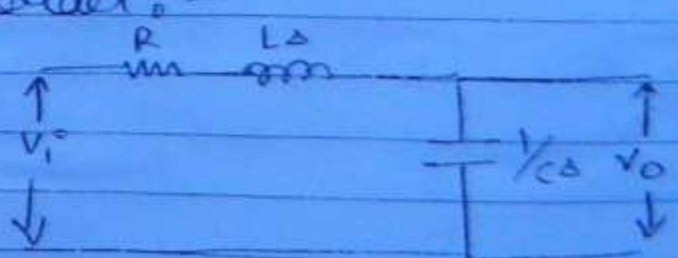




Eg:- RTD - Resistance Temp^r Detector.

Thermo Couple.

Second order :—



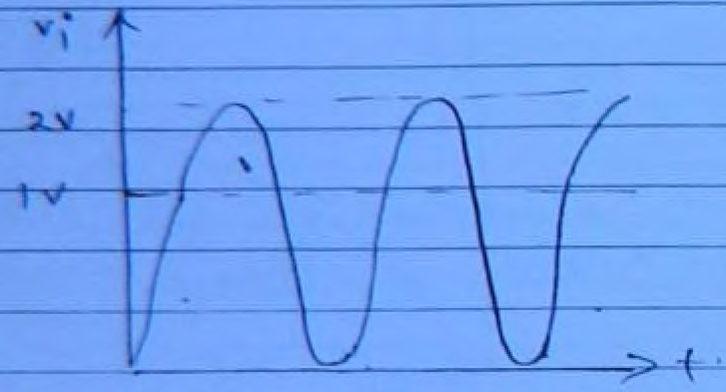
$$\frac{V_0}{V_i} = \frac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$$

(6)

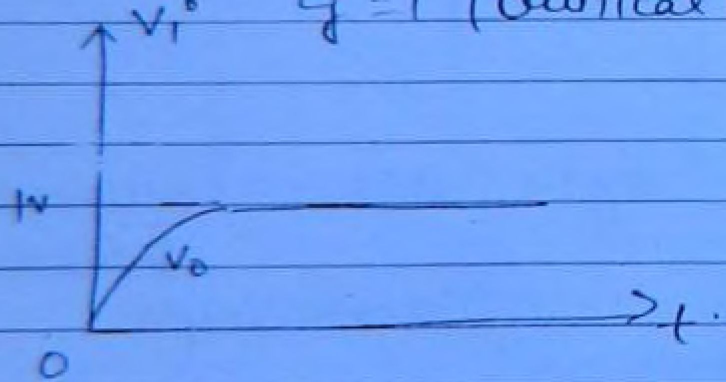
$$T(s) = \frac{V_0}{V_i} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ = Damping factor or (Ratio).

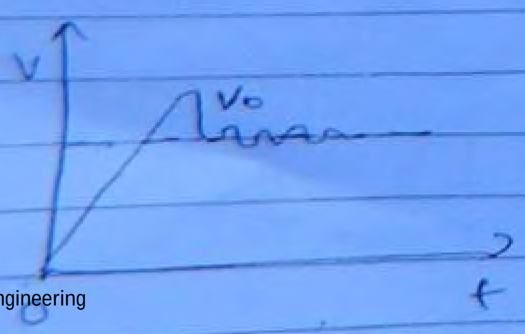
$\zeta = 0$ (undamped)



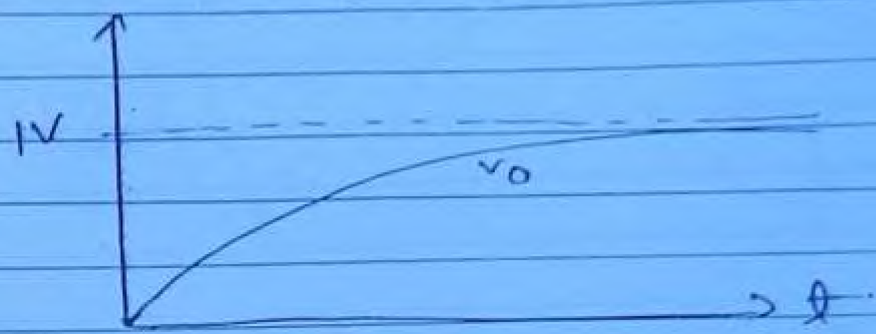
$\zeta = 1$ (Critical Damped)



$0 < \zeta < 1$ (underdamped)



$\delta > 1$ (overdamped)



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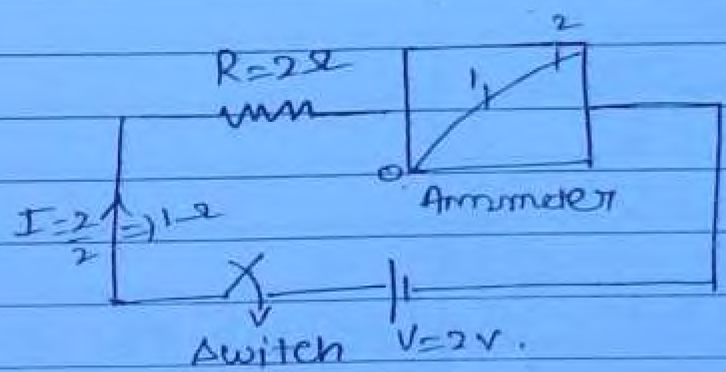
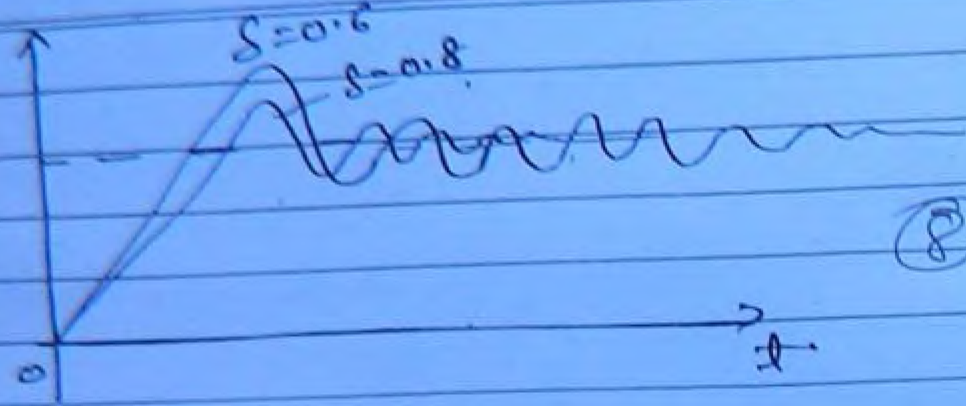


Indicating Meters:- Electrical system components (motor, generators, transmission line etc. are containing R, L, C parameter's and hence they are behaving like second order system to measure voltage, current and power the instrument should also behave similar to second order system. Hence the indicating meter are of second order type having a damping factor between 0.6 to 0.8. If the damping factor is nearer to the critical damped value than the output reaches to the study state value at faster rate in minimum time.

The time response of indicating meter is depend's on damping of the system.



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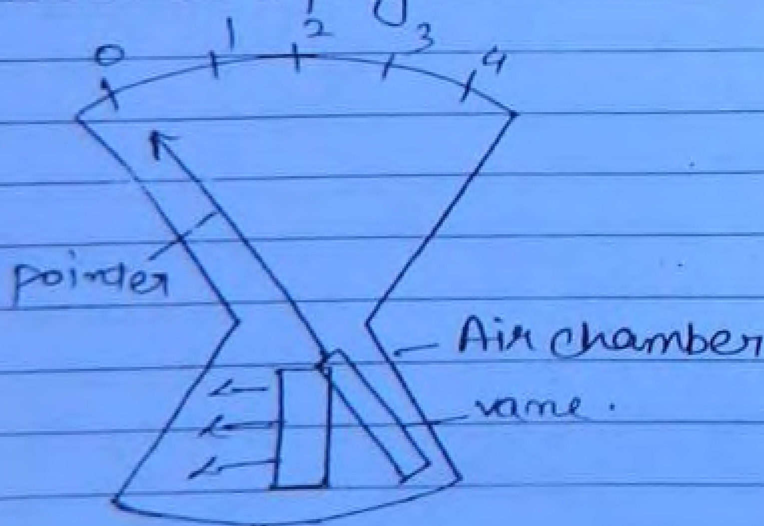
Torques of Indicating Instrument →

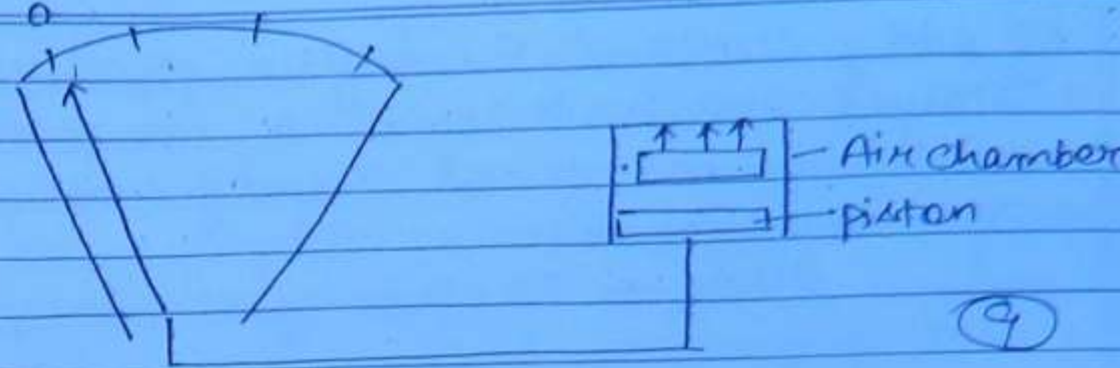
1. Damping Torque (T_D)
2. Deflecting Torque (T_d)
3. control Torque (T_c)

Damping Torque (T_D):- Used to stop the damping's produced at the final steady state of the instrument's.

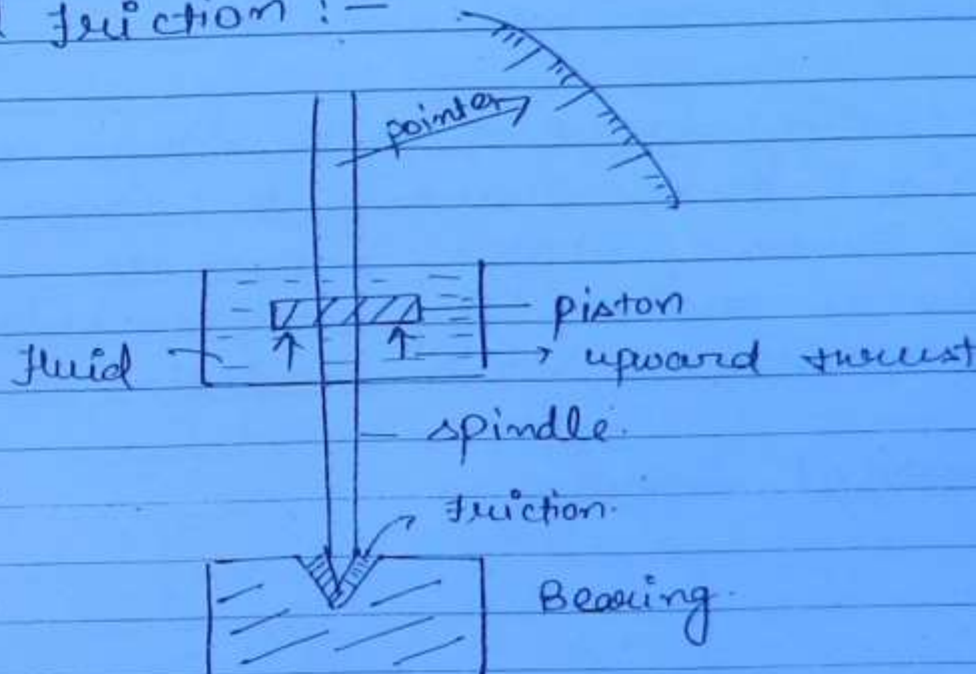
(i) Air Friction Damping:-



Eg :- → Moving Iron instrument's
→ Electrodynamometer.



(ii) Fluid friction :-



Example :- Electrostatic Meter.
Fluid friction is used where the deflecting torque is min^m. to reduce the friction b/w bearing and spindle and also used to reduce damping at final steady state position.

(iii) Eddy Current / Electromagnetic Damping :-

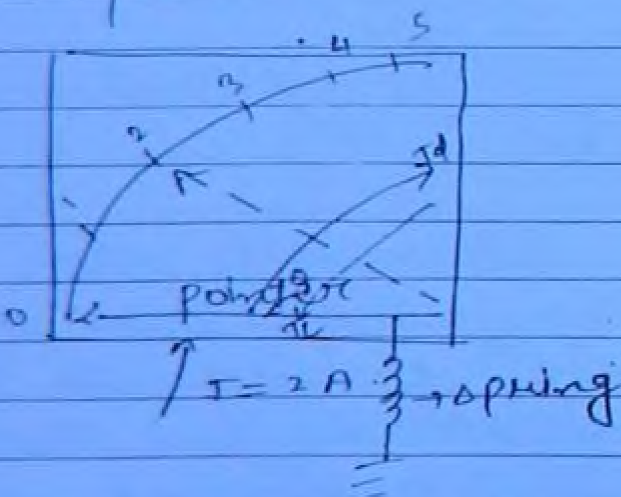
whenever a current carrying coil wound on a core and this core is moving in a magnetic field an EMF is induced. This will produce us an eddy current. The interaction

of eddy current and flux produces damping torque which is used to damp out the oscillation at final steady state position.
Eg:- Galvanometer – Eddy
PMMC — Electromagnetic damping.

It is used where the permanent magnet is used in the instruments.

2) Deflecting Torque → These torque are proportional to the quantity to be measured.
i.e Current, voltage, power etc
This is produced by using the principals of electromagnetic, Static induction etc.

3) Control Torque (T_c) →



At balance $T_d = T_c$

$T_c \propto \theta$

$$\boxed{T_c = K\theta}$$

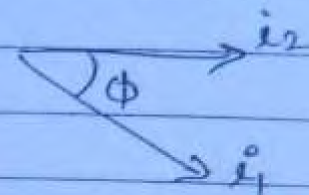
θ = deflecting angle in radians.
T_c = control torque in N·m.
K = spring constant in N·m/rad.

2) Voltmeter
3) wattmeter
4) powerfactor meter
5) Energymeter
6) Megger → High resistance measurement.
7) power frequency meter.

AC Measurement :-

$i_1 = I_{m_1} \sin(\omega t - \phi)$

$i_2 = I_{m_2} \sin \omega t$



$$T_{d\,avg} = \frac{1}{T}\int_0^T T_d \cdot dt = \frac{1}{T}\int_0^T \left(i_1 i_2 \frac{dM}{d\theta}\right) dt$$

for sinusoidal signal $T = 2\pi$

$$T_{d\,avg} = \frac{1}{2\pi}\int_0^{2\pi} I_{m_1} I_{m_2} \sin \omega t \cdot \sin(\omega t - \phi)\, d(\omega t) \frac{dM}{d\theta}$$

$$T_{d\,avg} = \frac{I_{m_1} I_{m_2}}{2} \cos\phi \frac{dM}{d\theta}$$

$$I_{RMS_1} = \frac{I_{m_1}}{\sqrt{2}} = I_1, \quad I_{RMS_2} = \frac{I_{m_2}}{\sqrt{2}} = I_2$$

$$T_{d\,avg} = I_1 I_2 \cos\phi \frac{dM}{d\theta}.$$

$T_c = K\theta.$

At balance $T_c = T_d$

$$K\theta = I_1 I_2 \cos\phi \cdot \frac{dM}{d\theta}$$

$$\boxed{\theta \propto I_1 I_2 \cos\phi}$$

wear and tear can be avoided and its life span is more.

(2) Temprature Error :- The internal resistance of the meter is proportional to surrounding temprature. If the temprature is changed than internal resistance will be changed so that the current flowing through the meter is affected and hence the meter read's wrong value.

It is compensated by low value of α material's like

Manganin — $\alpha = 0.00015/°C$

or

Constantan.

Copper — $\alpha \Rightarrow .004/°C$

(3) Frequency Error :- The inductance of the coil is depend's on frequency if the input frequency is changed than the inductive reactance will changed which causes error in the meter reading.

% Relative Limiting Error $= \varepsilon_r = \dfrac{\text{Measured value} - \text{True Value}}{\text{True Value}} \times 100$

$$\boxed{\% \varepsilon_r = \frac{A_m - A_t}{A_t} \times 100}$$

$R_t = 100\Omega$, $R_m = 105\Omega$

$$\% \varepsilon_r = \frac{105-100}{100} \times 100 \Rightarrow 5\%$$

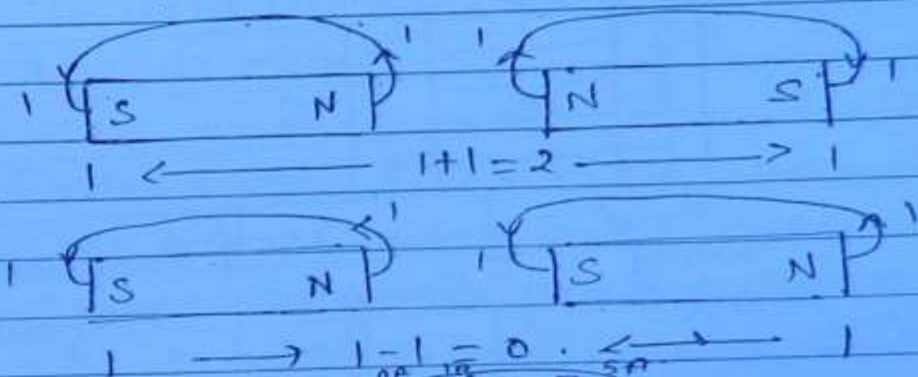
$R_m = 105\ \Omega$

$R_t = 100\ \Omega$

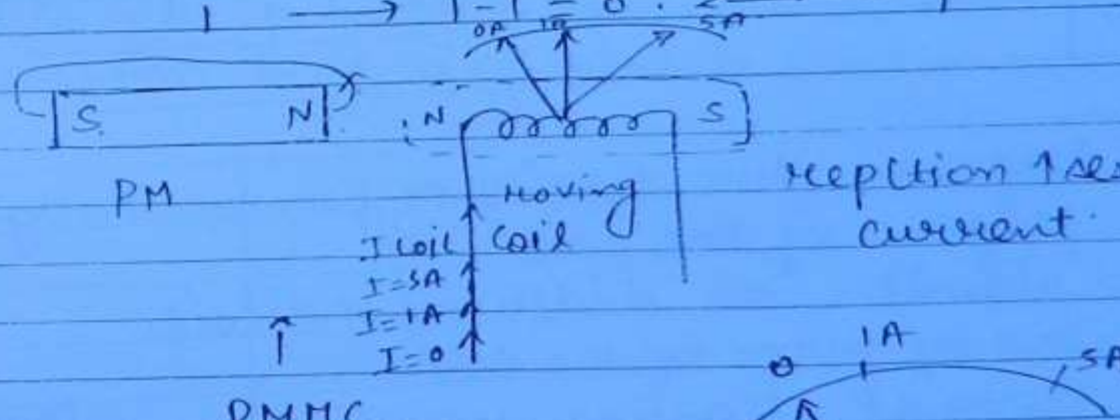
Error $= 5\ \Omega$

$$\boxed{\text{Accuracy} = -\text{Error}}$$

Electromagnetic Meters:-



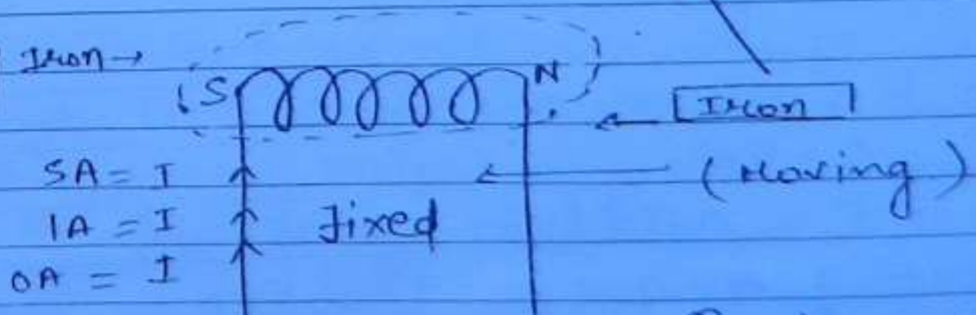
(1)



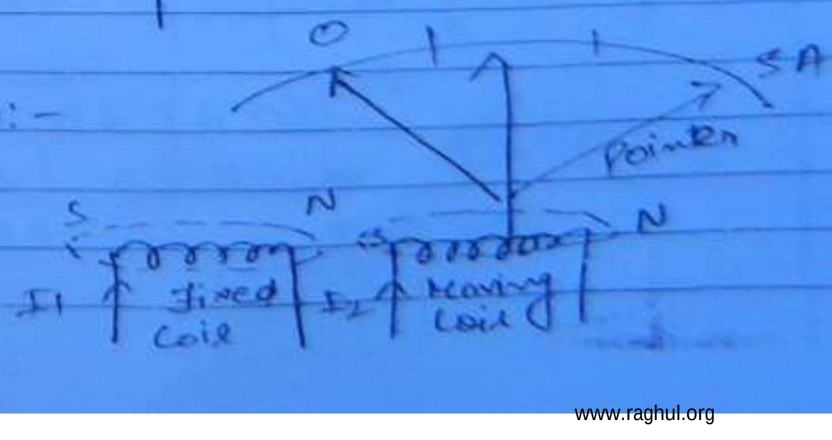
Repltion ↑ses due to current

PMMC

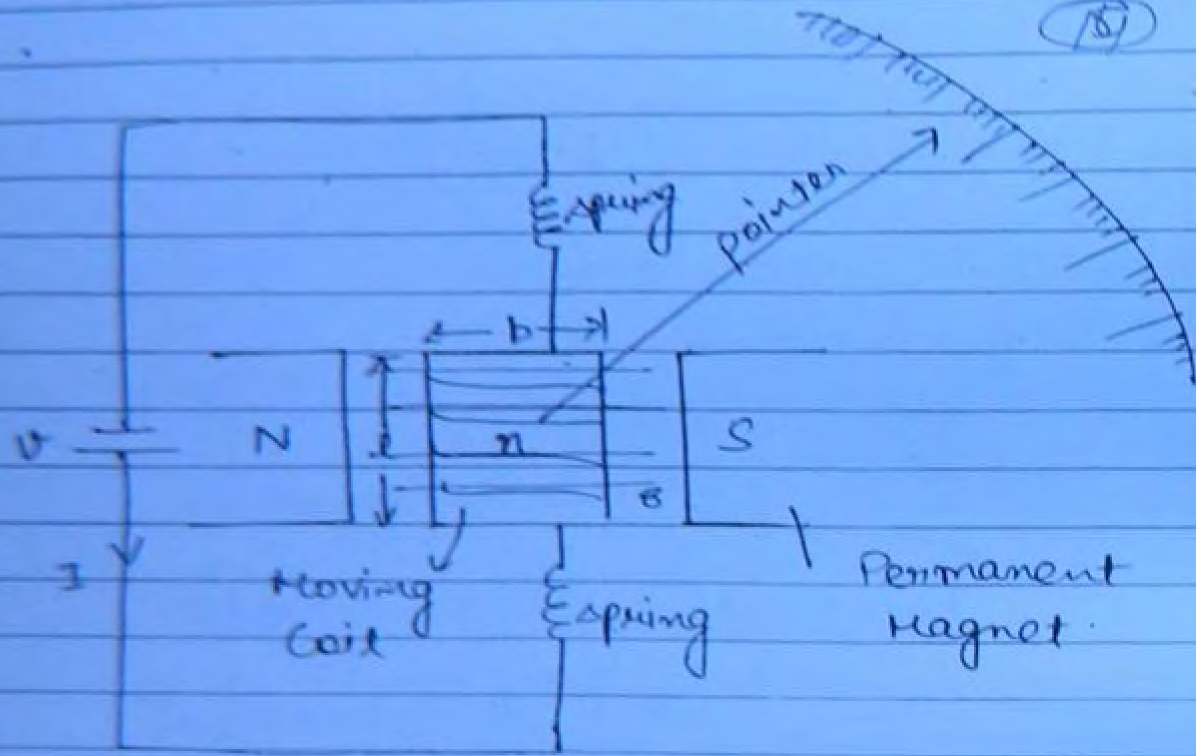
(ii) Moving Iron →



(3) Electrodynamometer:-



① PMMC [Permanent Magnet Moving Coil]:-



B = Magnetic flux density of permanent magnet.
l = Length of the coil
b = breadth or width of the coil.
n = no of turns of the coil.
I = Current flowing through Meter [current to be measured.]
A = Area of coil = lb

Force $F = nBIl\sin\theta$

$\theta = 90^\circ$

$F = nBIl$

Deflecting torque, $T_d = F \times b$

$T_d = nBIlb$

$\Rightarrow nBIA$

$T_d = GI$, $G = nBA$

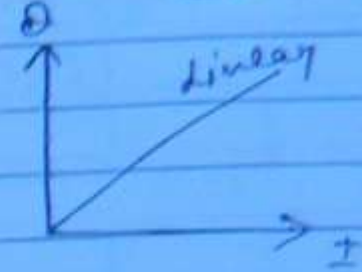
$\boxed{T_d \propto I}$

$T_C = K\theta$

At balance, $T_C = T_d$

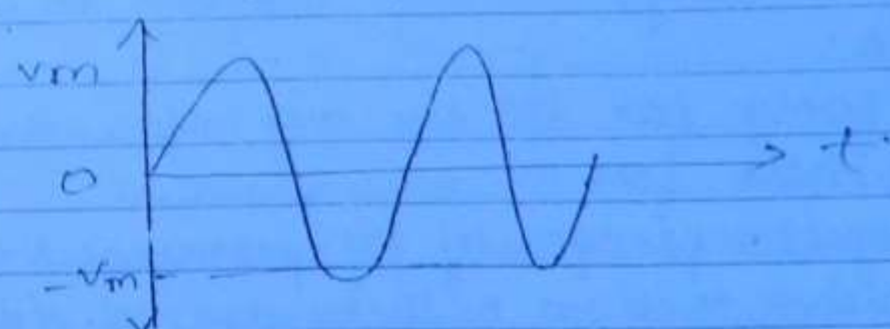
$K\theta = GI$

$\boxed{\theta \propto I}$

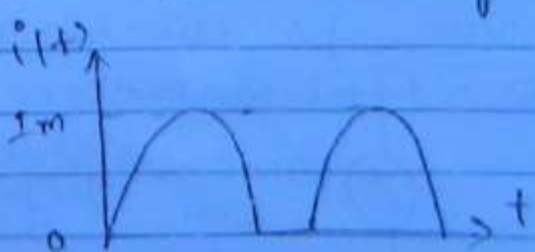


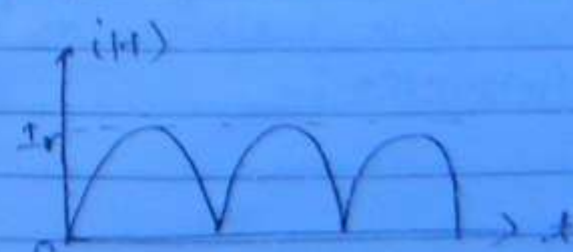
(1) Measures DC or Avg current or voltage.
(2) Scale is linear or uniform.
(3) Accuracy is more.

Note (i) PMMC measures DC or avg quantity.
(ii) If a ac signal with equal value magnitude of +ve & negative is passing through PMMC then the pointer vibrates nearer to the zero position bcoz avg value is zero.



(iii) If the output of half wave rectifier is passing through PMMC than it will read $I_{avg} = \frac{I_m}{\pi}$.
Similarly, the output of full wave rectifier PMMC reads, $I_{avg} = 2I_m/\pi$.



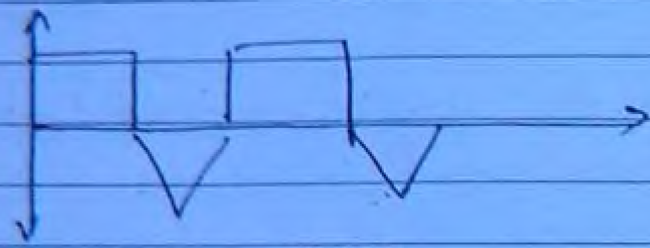


$I_{avg} = \frac{I_m}{\pi}$ $\qquad$ $I_{avg} = \frac{2I_m}{\pi}$

For any waveform find the Iavg by using

$$\boxed{I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt}$$

(16)



The current $i(t) = I_0 + I_1 \sin\omega t + I_2 \sin 2\omega t + \ldots$ is passing through PMMC it reads.

$$\boxed{I_{avg} = I_0}$$

Disadvantage.

1) Torque/weight ratio is higher in PMMC so that accuracy is more compare to other meter's.

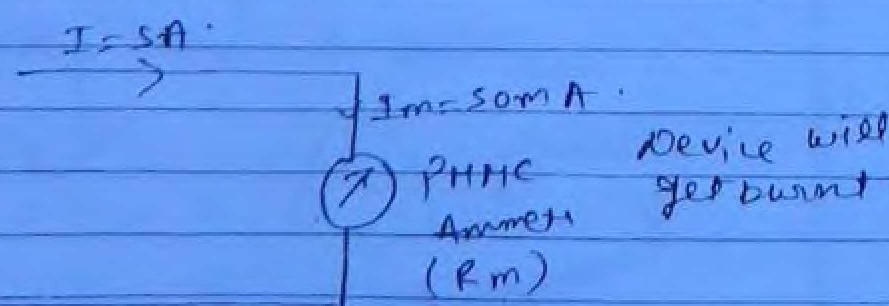
2) Used only for DC or avg measurement not suitable for AC measurement.

3) The magnetic property of permanent magnet and spring tension reduces due to surrounding temperature and aging.

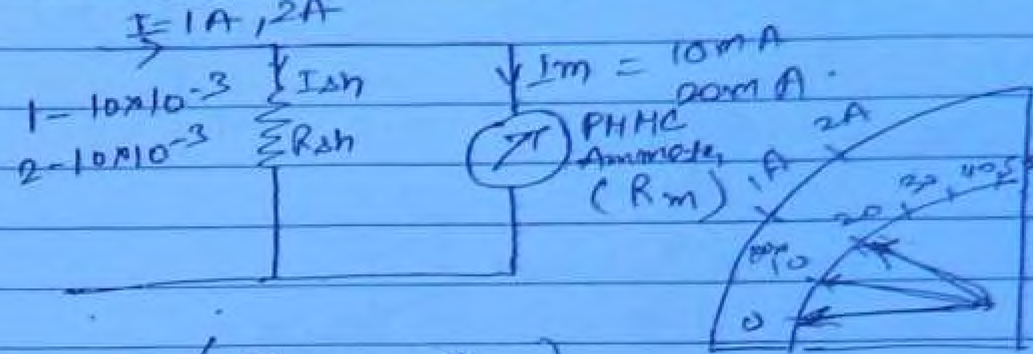
4) For direct measurement it is suitable to measure current upto 50mA or 100mV.

Enhancement of Meters:-

(1) Ammeter:-



Ammeter Shunt's :-



$(R_{sh} << R_m)$

$$m = \text{multiplication factor} = \frac{I}{I_m} = \frac{5}{50\times10^{-3}} \rightarrow 100$$

$$I_m = \frac{I \cdot R_{sh}}{R_{sh}+R_m}$$

$$\frac{R_{sh}+R_m}{R_{sh}} = \frac{I}{I_m} = m.$$

$$\boxed{R_{sh} = \frac{R_m}{(m-1)}}$$

For enhancment of ammeter, the shunt resistance (R_{sh}) is connected in parallel to the meter R_{sh} is made of magnenin which has low value of α.

[Voltage drop = $V_m \cdot R_m$]

Q A PMMC ammeter has a meter resistance of 100Ω and measure's current of 10 mA with direct measurent how much of shunt resistance is required to measure current's of

(i) 100 mA
(ii) 1 A
(iii) 10 A

Find the voltage drop across the meter at full

scale in each case?

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~~(I) 100 A.~~

$$~~R = \frac{10\times10^{-3}}{100}~~$$

$$~~= 10^{-4}.~~$$

$$m = \frac{100}{10\times10^{-3}} \Rightarrow 10,000$$

$$R_{sh} = \frac{R_m}{(m-1)}$$

(1) $I_m = 10\,mA$

$R_m = 100\,\Omega$

(i) $m = I_{ro}/I_m = 100/10 = 10$

$R_{sh} = R_m/(m-1) = 100/(10-1) = 11.1\,\Omega$

$V_m = I_m \cdot R_m = 10\times10^{-3}\times100 = 1V$

(ii) $I = 1\,A.$

$$m = \frac{1}{10\times10^{-3}} = 100$$

$$R_{sh} = \frac{100}{100-1} = 1.01\,\Omega$$

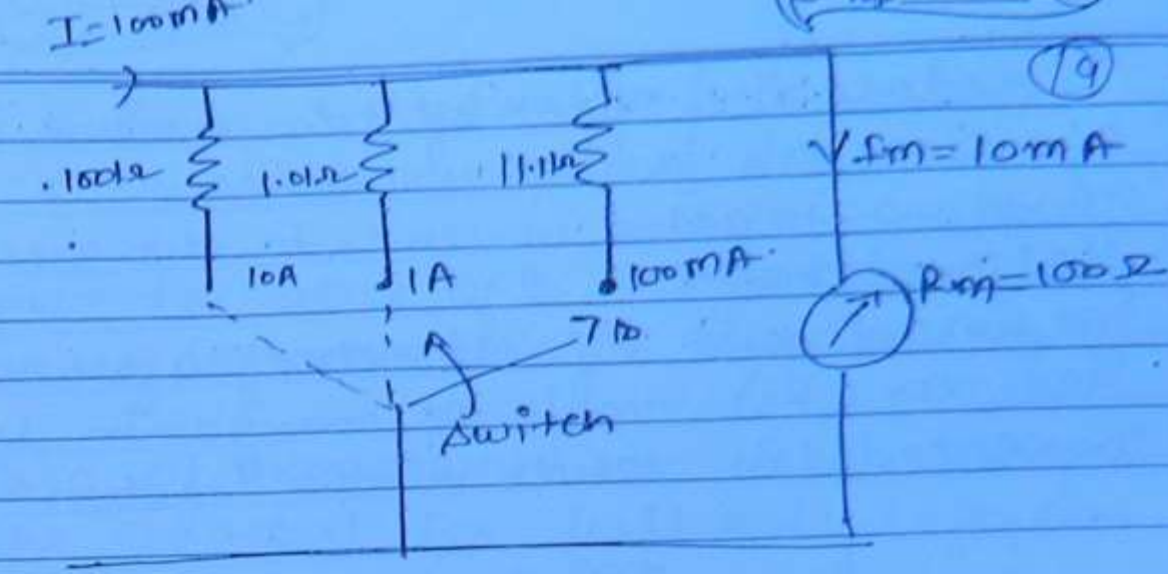
$V = I_m \cdot R_m = 1V$

(iii) $I = 10A$

$$m = \frac{10}{10\times10^{-3}} = 1000$$

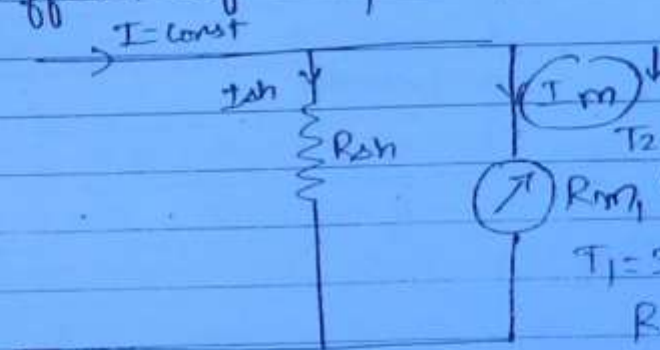
$$R_{sh} = \frac{100}{1000-1} = \cdot1001\,\Omega$$

$V = I \cdot R_m = 1V$



Multi Ammeter.

Effect of Temperature on Ammeter →



$\downarrow$ ≠ → Meter read wrong value.

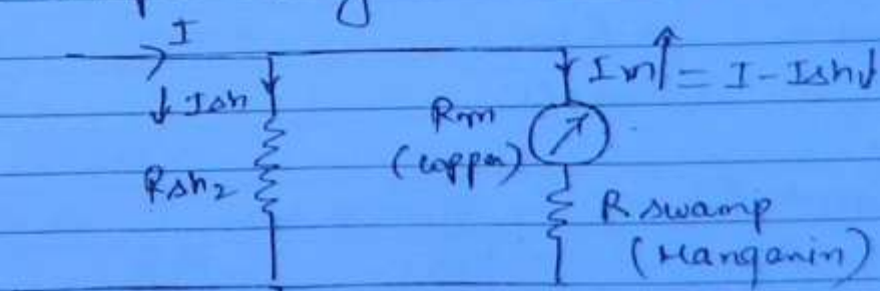
$T_2 = 50°C$ $R_{sh_1} = \dfrac{R_m}{(m-1)}$

$R_{m_1} \uparrow$ $R_{m_2} \uparrow$ R_{m_1}

$T_1 = 30°C$

$R_{m_2} = R_{m_1}[1 + \alpha(T_2 - T_1)]$

For compensating this we use →



$$I_m = \frac{I \cdot R_{sh_2}}{R_{sh_2} + R_{m_1} + R_{swamp}}, \quad m = I/I_m$$

$$R_{sh_2} = \frac{R_{m_1} + R_{swamp}}{(m-1)} = \frac{R_{m_1}}{(m-1)} + \frac{R_{swamp}}{(m-1)}$$

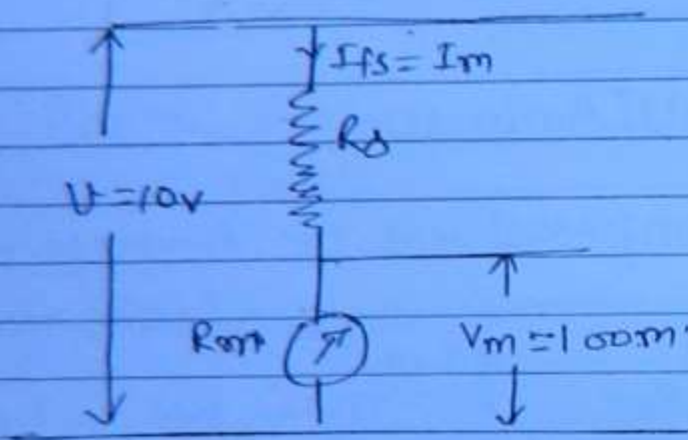
$$R_{sh_2} = R_{sh_1} + \frac{R_{swamp}}{(m-1)}$$

$R_{sh_2} > R_{sh_1}$

To reduce the effect of temprature in case of ammeter's swamp resistance made of manganin is added in series to the meter.

(ii) Voltmeter :-

For Enhancement of voltmeter's a series multiplier resistor (R_s) made of manganin is connected in series to the meter.



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$(R_s >> R_m)$

$$m = \frac{V}{V_m} = \text{Multiplication factor}$$

$$V_m = \frac{V \cdot R_m}{(R_m + R_s)}$$

$$\frac{R_m + R_s}{R_m} = \frac{V}{V_m} = m$$

$$\boxed{R_s = R_m (m-1)}$$

full scale Current $= I_{fs} = \dfrac{V}{(R_s + R_m)}$

Voltmeter sensitivity (Or) figure of Merit (FOM) :-

$$S_v = \frac{1}{I_{fs}} = \frac{R_s + R_m}{V} \quad \Omega/V$$ (21)

Note:- Higher the value of S_v, higher the sensitivity accuracy of meter. Lower the loading effect

→ In case of Ammeter which takes low value of the current for full scale deflection than its sensitivity is maximum.

Eg:- Two meter's A & B having a full scale current of 1 mA & 5 mA than meter A is more sensistive than meter B.

Q The voltmeter having an internal resistance of 100 Ω measures the voltage upto 10 mV without R_s. How much of R_s is required to measure a voltage of (i) 100 mV, (ii) 1 V (iii) 10 V. In Each case find full scale current and voltmeter sensitivity

As $R_m = 100\ \Omega$

$V_m = 10\ mV$.

(i) $V = 100\ mV$

$$m = \frac{V}{V_m} = 10$$

$$R_s = R_m(m-1)$$
$$= 100(10-1)$$
$$= 900\ \Omega$$

$$I_{fs} = \frac{V}{R_s + R_m} = \frac{100 \times 10^{-3}}{900 + 100} = 0.1\ mA$$

$$S_v = \frac{1}{I_{fs}} = \frac{1}{0.1} \Rightarrow 10\ k\Omega/V$$

(2) $V_1 = 1V$

$m = \frac{1}{10\times10^{-3}} = 100$

(22)

$R_s = 100\ (100-1)$
$= 9900\ \Omega$

$I_{fs} = \frac{1}{9900+100} = 0.1mA$

$S_V = 10\ K\Omega/V$.

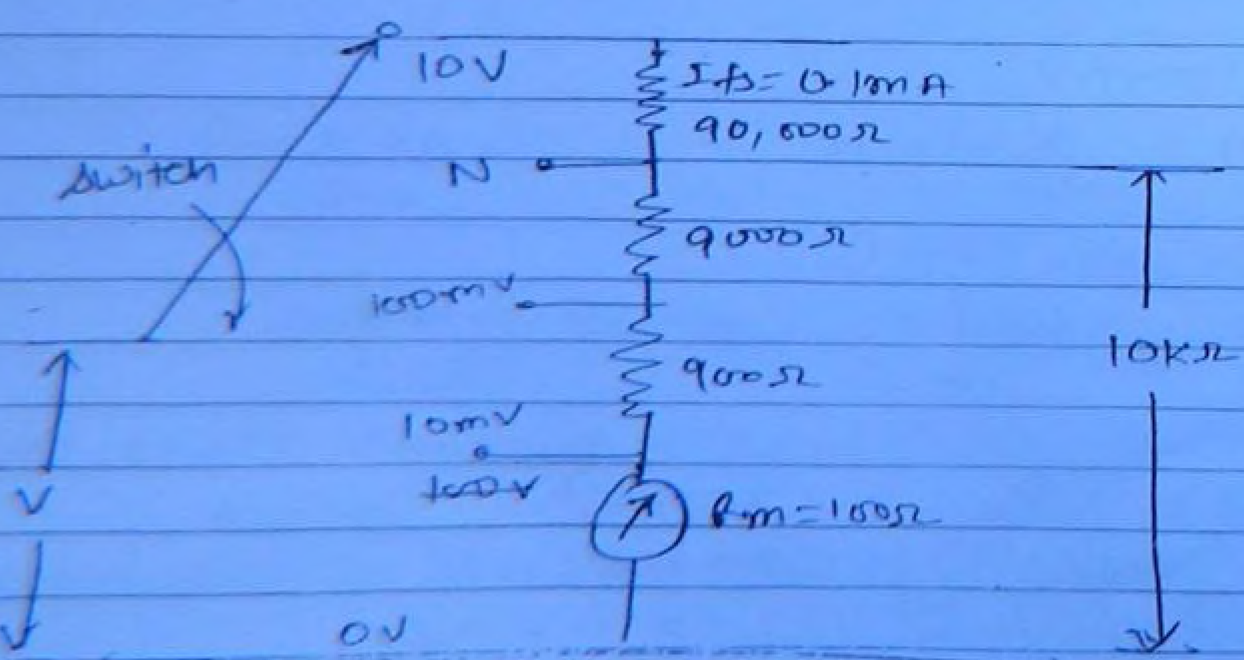
(3) $V = 10V$.

$m = \frac{10}{10\times10^{-3}} = 1000$

$R_s = 100\ (1000-1)$
$= 99900\ \Omega$

$I_{fs} = \frac{10}{99900+100} = 0.1mA$

$S_V = 10K\Omega/V$.



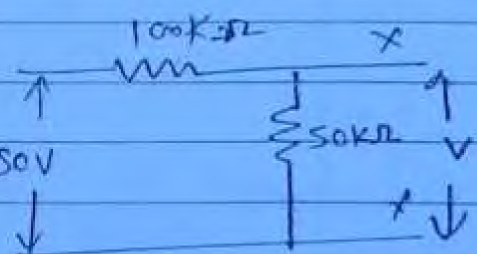
Multi range Voltmeter

Voltmeter Sensitivity indicates the change in the resistance for unit change in the voltage.

Loading Effect :- If a low sensitivity voltmeter's are used for measurement of voltage across equipment the meter will work's as a load and the current is passing through the meter so that instrument read wrong value. This is called loading effect.
To reduce the loading effect high sensitivity voltmeter's are to be used.

Q. Two voltmeter's A & B are used to measure voltage across 15 kΩ resistance for the circuit shown in the figure. Meter A has sensitivity of 1 kΩ/v and Meter B has sensitivity of 1 MΩ/v. Find the error in the measurement of voltage across the 50 kΩ.

As



$V_{True} = V_T = \frac{50}{100+50} \times 150$

$\Rightarrow 50V.$

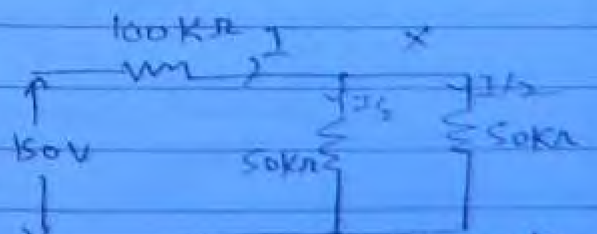
eter A

$S_V = 1 k\Omega/V$

$V = (0-50)V$

$R_S + R_m = V S_V$

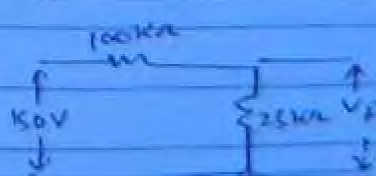
$\Rightarrow 50 \times 1 \Rightarrow 50 k\Omega$



$50 \| 50$
25

Measured $- V_A = \frac{25 \times 150}{100+25}$

$\Rightarrow 30V.$



$$\% \varepsilon_r = \frac{V_m - V_T}{V_T} \times 100$$

$$\Rightarrow \frac{30-50}{50} \times 100$$

24

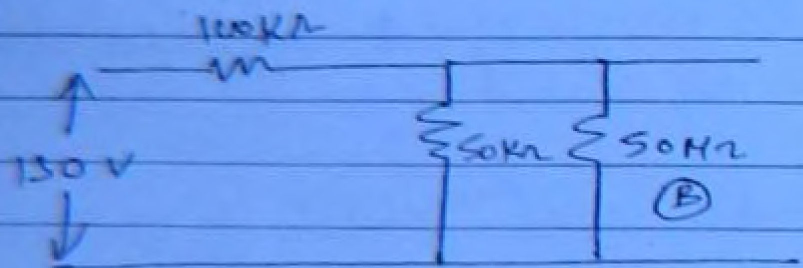
$$\boxed{\varepsilon_r = -40\%}$$

B

$S_v = 1 M\Omega/V$

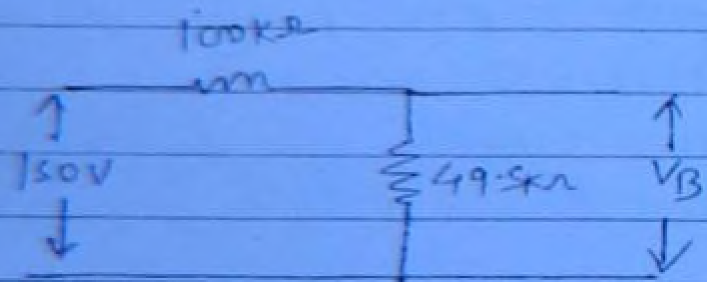
$V = 0\text{-}50V$

$R_s + R_m = V S_v = 50 M\Omega$



$50K \parallel 50M\Omega$

$\Rightarrow 49.5 K\Omega.$

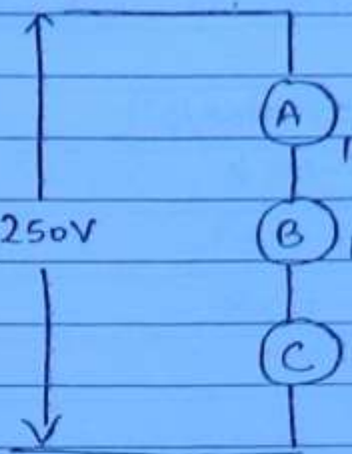


$$V_B = \frac{150 \times 49.5}{100 + 49.5} = 49.7V$$

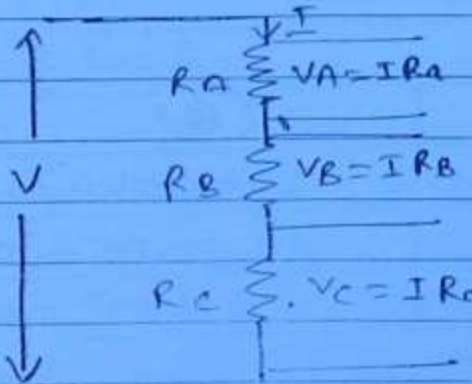
$$\varepsilon_r = \frac{49.7 - 50}{50} \times 100$$

$$\boxed{\varepsilon_r = -0.6\%}$$

Voltmeter with different Ratings:-



Given Rating	A	B	C
	V, S_V	I_{fs}, V	$I_{fs}, (R_s+R_m)$
	$S_V = \frac{R_s+R_m}{V}$	$I_{fs} = \frac{V}{R_s+R_m}$	$R_C = (R_s+R_m)$
	$R_A = R_s+R_m$ $= V S_V$	$(R_s+R_m) = \frac{V}{I_{fs}}$ $= R_B$	



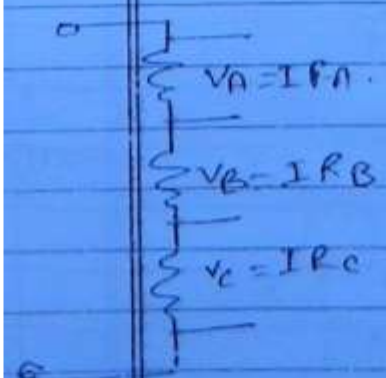
$$I = \frac{V}{(R_A+R_B+R_C)}$$

32
Q17

A	B	C
100V, 5mA	100V, 250Ω/V	10mA, 15KΩ
$R_A = \frac{100}{5\times10^{-3}} = 20k\Omega$	$R_B = 100\times250$ $\Rightarrow 25K\Omega$	$R_C = 15K\Omega$

$100\times5\times10^{-3}$
0.5
250
100 → 150
120
4,00×5

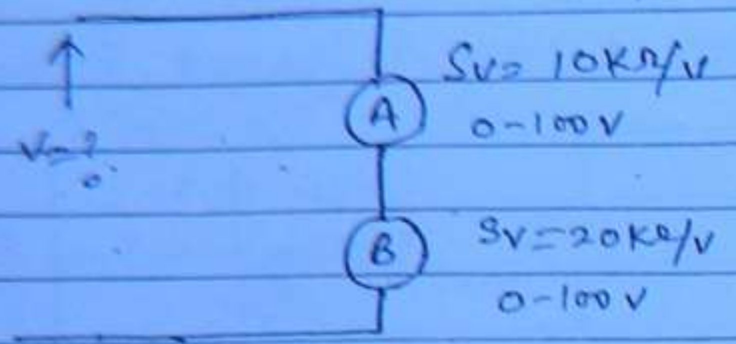
$$I = \frac{120}{20+25+15} \Rightarrow 2\ mA.$$

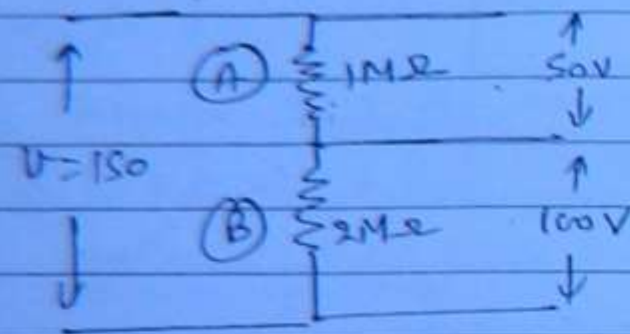


$V_A = 2\times20 = 40V$

$V_B = 25\times2 = 50V$

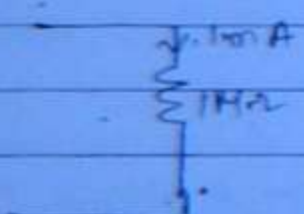
$V_C = 15\times2 = 30V$

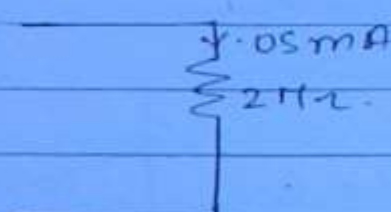


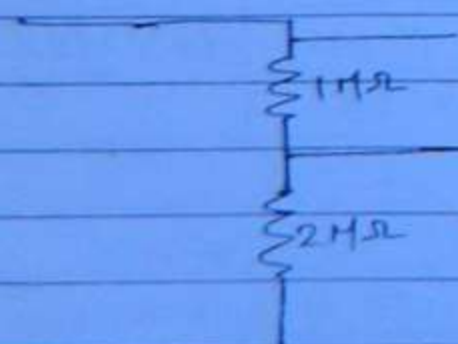


OR

A	B
$S_V = 10K\Omega/V$	$S_V = 20K\Omega/V$
$I_{fs} = \frac{1}{S_V} = 0.1mA$	$I_{fs} = .05mA$







$VA = 1\times10^{6} \times .05\times10^{-3}$
$= 50V$

$VB = 2\times10^{6} \times .05\times10^{-3}$
$= 100V.$

Q32 $V = 50V$.

$S_V = 5K\Omega/V$, $I_{fs} = 200\mu A$ (97)

$R_m = 100\ \Omega$

$R_s + R_m = V S_V \Rightarrow 50 \times 5 = 250 K\Omega$

$R_s + 100 = 250 \times 10^3$

$R_s + .1 K\Omega = 250 K\Omega$

$R_s = 249.9\ K\Omega$.

Q33 $I_{fs} = 50\mu A$

$V = 10V$

$R_m = 500\ \Omega$

$R_s + R_m = \frac{V}{I_{fs}} = \frac{10}{50 \times 10^{-6}} \Rightarrow 199.5 K\Omega$

Q34 $I_{fs} = 200\ \mu A$.

$S_V = \frac{1}{200 \times 10^{-6}} \Rightarrow 5K\Omega/V$

$\Rightarrow 5\Omega/mV$

Q37 $I_m = 1mA$

$R_m = 100\Omega$

$I = 100mA$

$m = \frac{I}{I_m} = 100$

$R_{sh} = \frac{R_m}{m-1} = \frac{100}{99} = 1.01\Omega$

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Q10 $T_c = K\theta$ 0.2% more

$K\downarrow = .04\% / ^\circ c \rightarrow 10^\circ c \Rightarrow K\downarrow = 0.4\%$ $T_c \downarrow = 0.4\%$

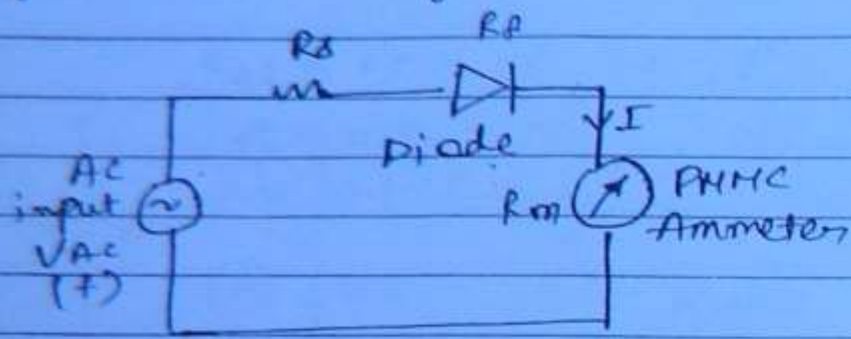
$T_d = nBIA$,

$B\downarrow = .02\% / ^\circ c \Rightarrow 10^\circ c \rightarrow B\downarrow = 0.2\%$ T_d 0.2%

Application of PMMC :-

Rectifier Meters:-

① Half wave Rectifier Meter:-



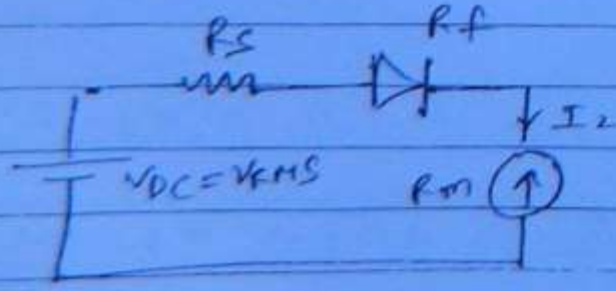
$$V_{AC} = V_m \sin\omega t.$$
$$= \sqrt{2}\, V_{RMS} \sin\omega t.$$

$$I = I_{avg} = \frac{V_{avg}}{(R_s + R_f + R_m)}$$
$$\Rightarrow \frac{V_m}{\pi(R_s + R_f + R_m)}$$
$$\Rightarrow \frac{\sqrt{2}\, V_{RMS}}{\pi(R_s + R_f + R_m)}$$

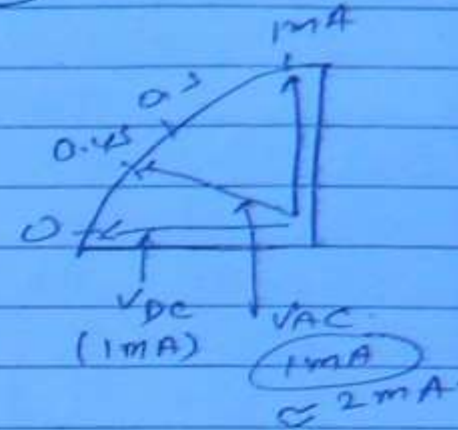
$$\left[I_1 = I_{avg} = \frac{0.45\, V_{RMS}}{(R_s + R_f + R_m)} \right] \quad \text{— ①}$$

Ideal Diode
$\boxed{R_f = 0}$

With DC input :-



$$I_2 = \frac{V_{RMS}}{(R_s + R_f + R_m)} \quad -(2)$$



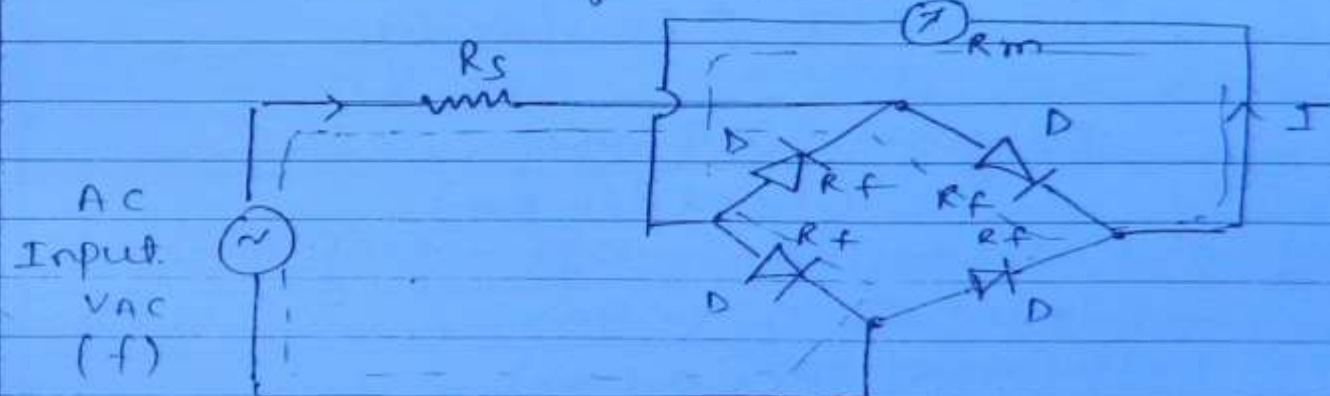
from (1) & (2)

$$\boxed{I_1 = 0.45 I_2}$$

$$\boxed{S_i ac = 0.45 S_i dc}$$

→ S_i = current sensitivity.

(2) Full wave Rectifier Meter:-



$$V_{AC} = V_m \sin \omega t.$$

$$= \sqrt{2} V_{RMS} \sin \omega t.$$

$$I_1, I_{avg} = \frac{V_{avg}}{(R_s + 2R_f + R_m)}$$

$$\Rightarrow \frac{2V_m}{\pi(R_s + 2R_f + R_m)}$$

$$\Rightarrow \frac{2\sqrt{2} V_{RMS}}{\pi(R_s + 2R_f + R_m)}$$

$$I_1 = I_{avg} = \frac{0.9 V_{RMS}}{(R_s + 2R_f + R_m)} \quad -(1)$$

With DC input →

$$I_2 = \frac{V_{RMS}}{(R_s + 2R_f + R_m)} \quad \text{--- (2)}$$

from (1) and (2)

$$\boxed{I_1 = 0.9\, I_2}$$

$$\boxed{S_i\, ac = 0.9\, S_i\, dc}$$

→ S_i = Current sensitivity

$$I_{avg} = \frac{0.9\, V_{RMS}}{(R_s + 2R_f + R_m)}$$

$$1 \times 10^{-3} \Rightarrow \frac{0.9 \times 100}{(R_s + 0 + 100)}$$

$$R_s + 100 = 90 \times 10^3$$

$$R_s = 90 \times 10^3 - 100$$

$$\Rightarrow 89.9\, k\Omega$$

$$I_{avg} = \frac{0.9\, V_{RMS}}{(R_s + 2R_f + R_m)}$$

Ignoring R_f & R_m.

$$45 \times 10^{-3} = \frac{0.9 \times 100 \times 10^3}{R_s}$$

$$R_s = 2 \times 10^6\, \Omega$$

$$X_c = \frac{1}{2\pi f c}$$

$$C = \frac{7}{\ldots}$$

Error due to Calibration → If a meter is Calibrated for a particular waveform eg:- sinusoidal .. RMS & Avg measurement. 31

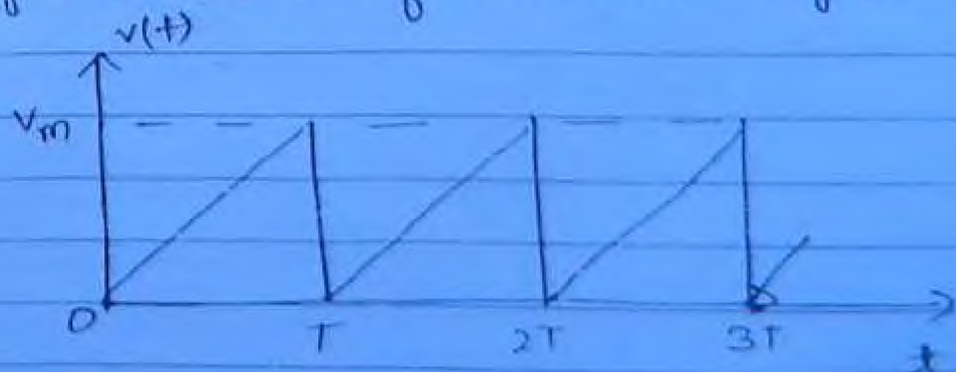
If the same meter is applied with other than sinusoidal waveform's. A meter produces error. this is called error due to calibration For calculation of these type of error form factor of origional waveform (sinusoidal) And the form factor of other than sinusoidal waveform are calculated from these values error is calculated.

$$\% \text{ Error} = E_r = \frac{\text{Measured} - \text{Time}}{\text{Time}} \times 100$$

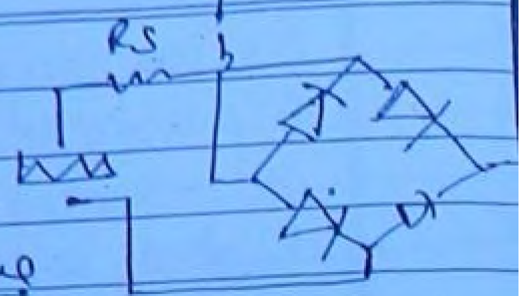
$$\Rightarrow \frac{\text{Form factor of origional (sinusoidal)} - \text{form factor of other signal}}{\text{form factor of other signal}} \times 100$$

Q. A full wave rectifier meter is calibrated to measure the RMS quantity of the sinusoidal input. For this meter a sawtooth waveform having a peak value of V_m & a time period of T is applied. Find the error in the measurement of RMS value of sawtooth signal.

As



Sinusoidal signal



$$\frac{I_{RMS}}{I_{avg}} = 1.11 = \text{form factor} = \text{Measured value}$$

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Saw tooth :-

$$V_{RMS} = \left[\frac{1}{T} \int_0^t v^2(t)\, dt \right]^{1/2}$$

$$v(t) = \frac{V_m}{T} t$$

$$V_{RMS} = \left[\frac{1}{T} \int_0^T \frac{V_m^2 t^2}{T^2}\, dt \right]^{1/2}$$

$$\Rightarrow V_m / \sqrt{3}$$

$$V_{avg} = \frac{1}{T} \int_0^T v(t)\, dt = \frac{1}{T} \int_0^T \frac{V_m}{T} t\, dt$$

$$\Rightarrow V_m / 2$$

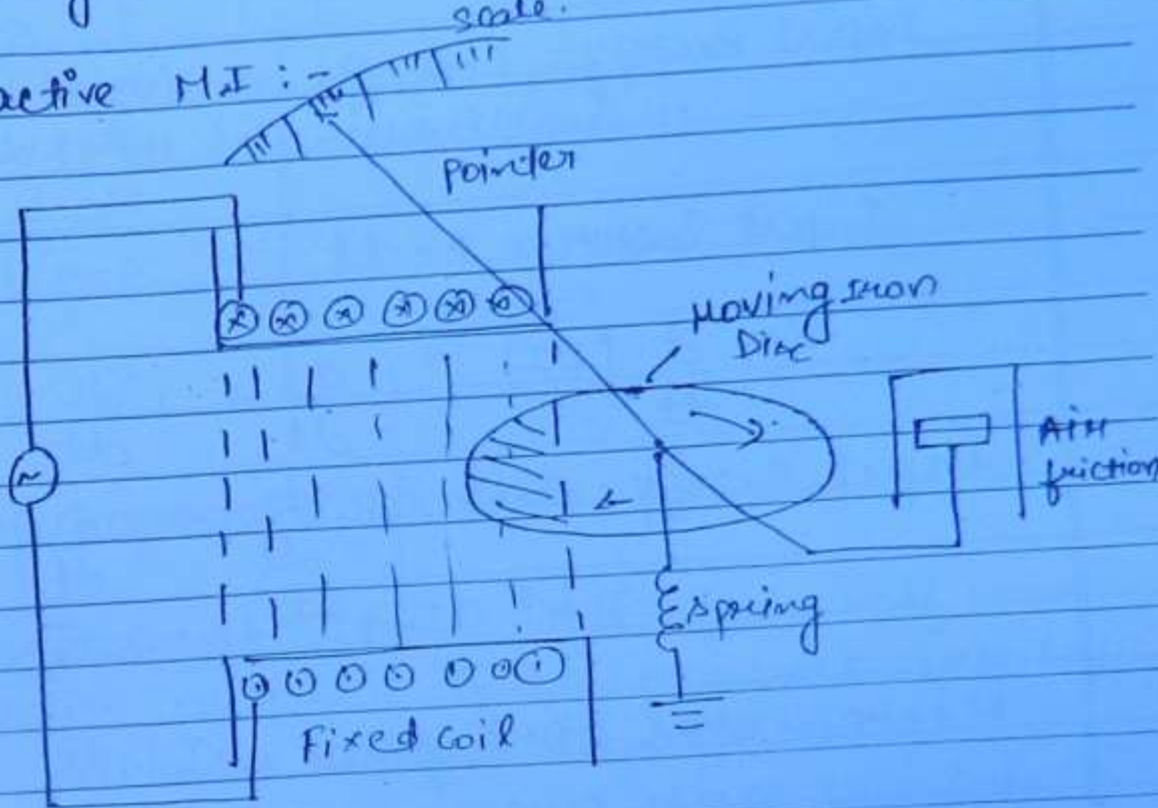
$$\text{form factor} = \frac{V_{RMS}}{V_{avg}} = \frac{V_m / \sqrt{3}}{V_m / \sqrt{2}} \Rightarrow 1.154 = \text{True}$$

$$\% \text{ Error} = \frac{1.11 - 1.154}{1.154} \times 100$$

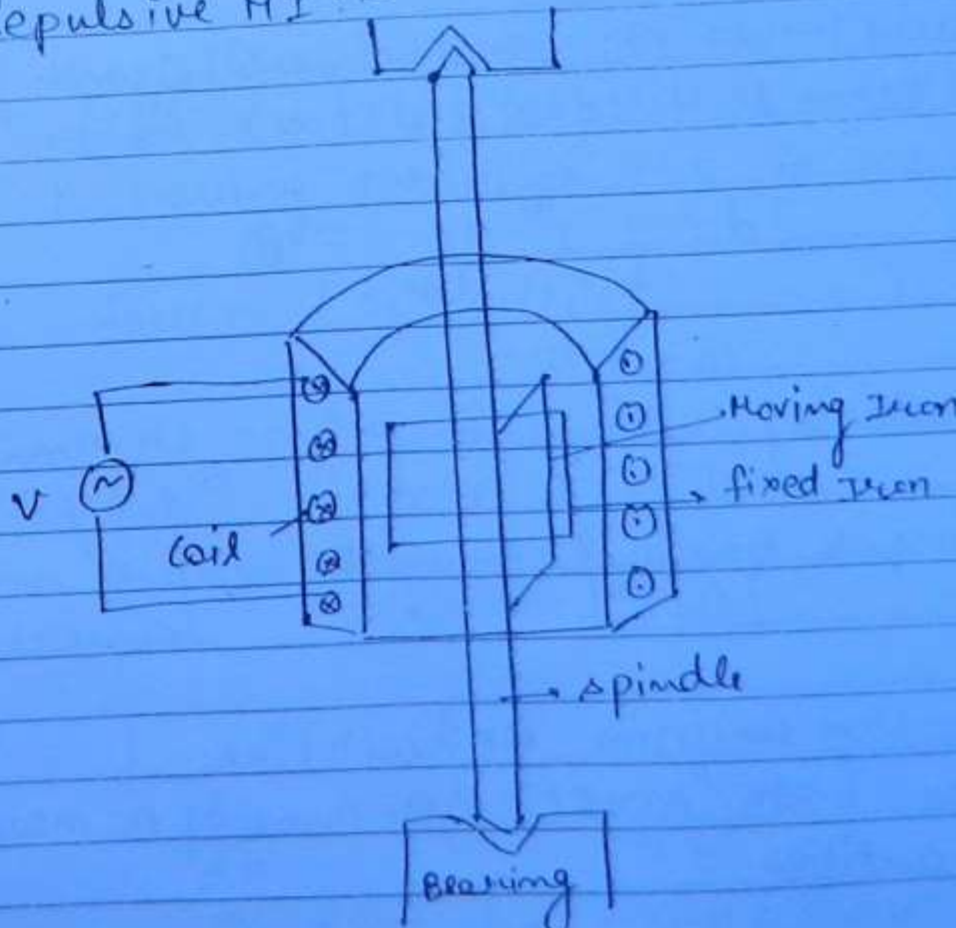
$$\Rightarrow -3.89\%$$

Moving Im Meter:- [MI]

① Attractive MI :-



② Repulsive MI :-



Deflecting Torque (Td)

Input Energy = change in stored Energy + Mechanical work done. → (1)

Input Energy = $V I \, dt$

$$V = \frac{d}{dt}[LI] = L\frac{dI}{dt} + I\frac{dL}{dt}$$

$$\text{Input Energy} = I\,dt\left[L\frac{dI}{dt} + I\frac{dL}{dt}\right]$$

$$\Rightarrow LI\,dI + I^2 dL \quad - (2)$$

Mechanical work done = $Td \cdot d\theta$ — (3)

$$\text{Change in stored Energy} = \frac{1}{2}[I + dI]^2 (L + dL) - \frac{1}{2} L I^2$$

substitute (2), (3) & (4) in (1) Igone. terms $dL(dI)^2$, $dI(dL)$ terms.

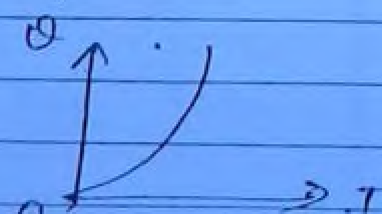
$$Td = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad - (4)$$

$$T_c = K\theta$$

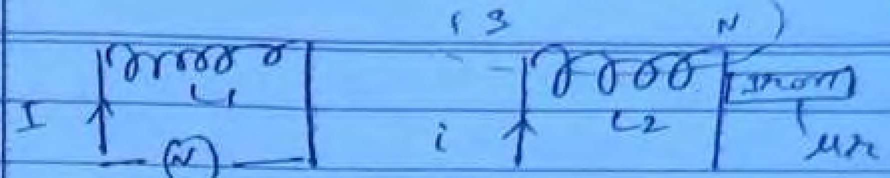
At balance, $T_c = Td$

$$K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\boxed{\theta \propto I^2}$$



Scale is Non uniform or non linear.
Measures both AC & DC. In case of AC measures RMS quantities.



$V = L \cdot \frac{dI}{dt}$

$L_1 = \frac{N^2 \mu_0 A}{l}$

$\frac{1}{2} L_1 \; I_1^2$

$L_2 = \frac{N^2 \mu_0 A \mu_r}{l}$

$\Rightarrow L_1 \mu_r$

$\frac{1}{2} L_2 I^2$

Some Important points →

(i) MI measures $I_{RMS} = I_m/2$ for a half wave rectifier output. And it measure $I_{RMS} = I_m/\sqrt{2}$ for full wave rectifier output.

For any other waveform shape calculate

$$\left[I_{RMS} = \left[\frac{1}{T} \int_0^T i^2(t)\, dt \right]^{1/2} \right]$$

If the current $i(t)$

$$i(t) = I_0 + I_1 \sin \omega t + I_2 \sin 2\omega t + \cdots$$

is passing through MI it measures

$$I_{RMS} = \sqrt{I_0^2 + \frac{1}{2}\left(I_1^2 + I_2^2 + \cdots\right)}$$

(ii) Control spring is used to produce controlling torque.

(iii) Air friction damping is used to produce damping torque.

Ch-2 Q6

$K = 25 \times 10^{-6}$.

$$\frac{dL}{d\theta} = \left(3 - \frac{2\theta}{4 \cdot 2}\right) \times 10^{-6}$$

$$K\theta = \frac{1}{2} I^2 \left(3 - \theta/2\right) \times 10^{-6}$$

$$K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$25\times10^{-6} = \frac{1}{2}\times(5)^2\times\left(3-2\theta/4\right)\times10^{-6}$$

$$\boxed{\theta = 1.2}$$

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$$I_{RMS} = \sqrt{I_0^2+\frac{1}{2}\left(I_1^2+I_2^2+ - -\right)}$$

$$\Rightarrow \sqrt{(-8)^2+\frac{1}{2}(6\sqrt{2})^2}$$

$$\Rightarrow 10$$

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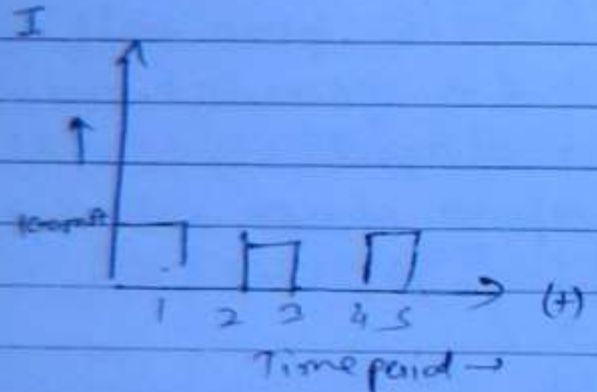


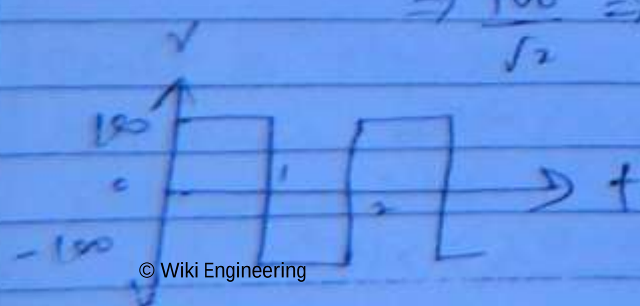
$$I(t) = 100\,mA, \qquad 0<t<1$$
$$= 0, \qquad 1<t<2$$

$$T = 2\,msec$$

$$I_{RMS} = \left[\frac{1}{T}\int_0^T I^2(t)\,dt\right]^{1/2}$$

$$\Rightarrow \left[\frac{1}{2}\int_0^1 (100)^2\,dt + \int_1^2 0.\,dt\right]^{1/2}$$

$$\Rightarrow \frac{100}{\sqrt{2}} \Rightarrow 70.7\,mA.$$



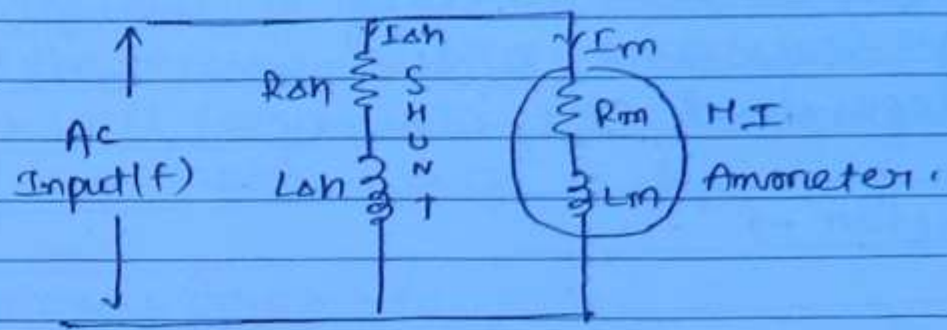
$I_{RMS} = 100\,mA.$

Error of M.I

Frequency Errors:-

(32)

(1) Ammeter:-



$$I_m = \frac{I \cdot (R_{sh} + j\omega L_{sh})}{R_m + R_{sh} + j\omega(L_m + L_{sh})}$$

$$\boxed{|I_m| = \frac{I\sqrt{R_{sh}^2 + \omega^2 L_{sh}^2}}{\sqrt{(R_m+R_{sh})^2 + \omega^2(L_m+L_{sh})^2}}} \quad -(1)$$

frequency Dependent.

$$I_{sh} = \frac{I \cdot \sqrt{R_m^2 + \omega^2 L_m^2}}{\sqrt{(R_m+R_{sh})^2 + \omega^2(L_m+L_{sh})^2}} \quad -(2)$$

$$\frac{(1)}{(2)} = \frac{I_m}{I_{sh}} = \sqrt{\frac{R_{sh}^2 + \omega^2 L_{sh}^2}{R_m^2 + \omega^2 L_m^2}}$$

$$\Rightarrow \frac{R_{sh}}{R_m}\sqrt{\frac{1+\omega^2\left(\frac{L_{sh}}{R_{sh}}\right)^2}{1+\omega^2\left(\frac{L_m}{R_m}\right)^2}}$$

If $\boxed{\frac{L_{sh}}{R_{sh}} = \frac{L_m}{R_m}}$; $\boxed{\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}}$

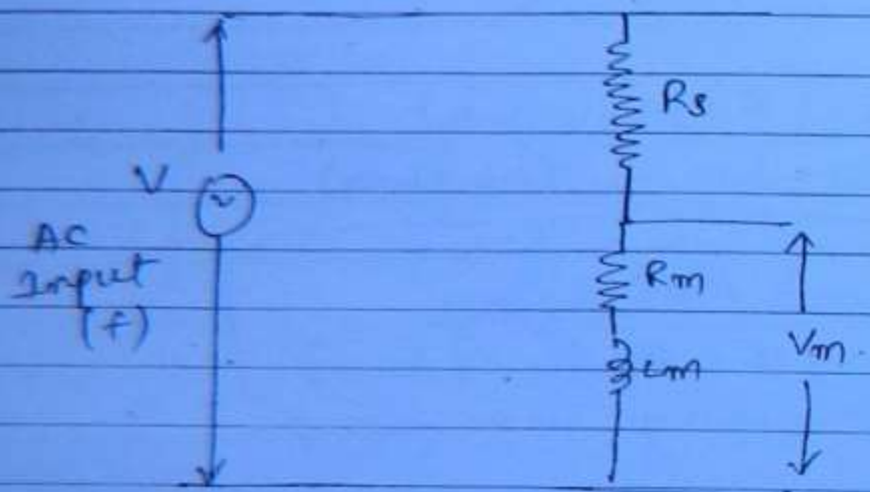
Independent of frequency.

(38) $i_1 = 30 \sin 2\pi \times 50t$, $I_{RMS1} = \frac{30}{\sqrt{2}}$, $f_1 = 50$.

$i_2 = 30 \sin 2\pi \times 100t$, $I_{RMS2} = \frac{30}{\sqrt{2}}$, $f_2 = 100$

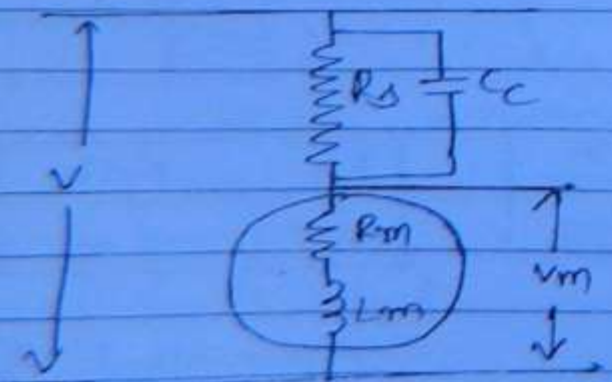
Note → If the time constant of the shunt is equal to time constant of the meter. Than ammeter is independent of frequency.

(2) Voltmeter →



Rs is very high, So its internal inductance neglected.

$$\boxed{V_m = \frac{V\sqrt{R_m^2 + \omega^2 L_m^2}}{\sqrt{(R_s+R_m)^2 + \omega^2 L_m^2}}} \rightarrow \text{Depends on frequency.}$$

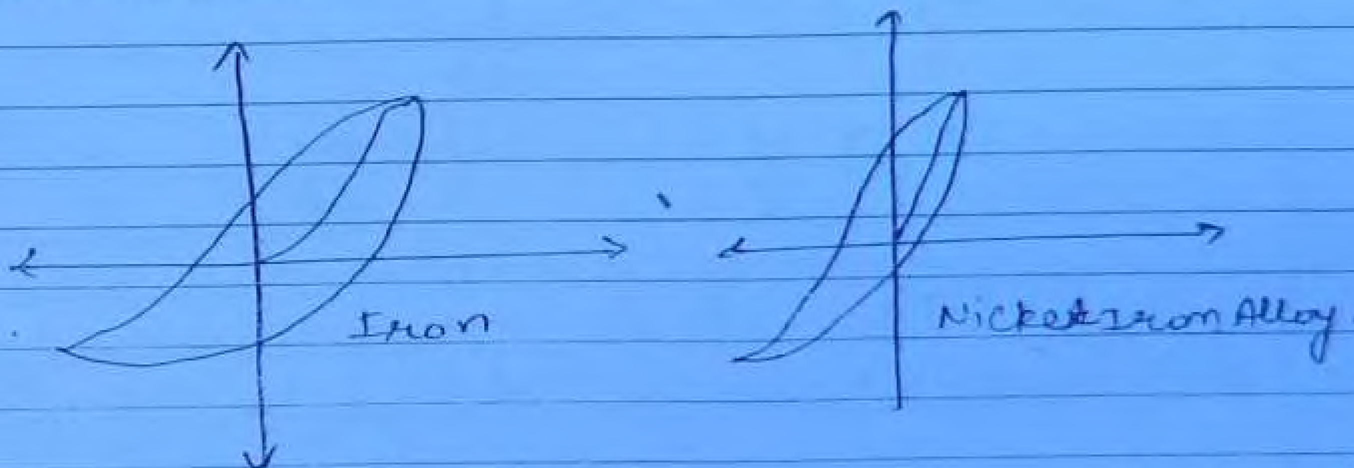


C_c = Compensating Capacitor.

$$\boxed{C_c = \frac{0.41\, L_m}{R_s^2}} \;;\; \boxed{C_c R_s = \frac{0.41\, L_m}{R_s}}$$

To make voltmeter independent of frequency a compensating capacitor is connected in parallel to the series multiply resistance R_s.
This will compensate the inductive reactance of the meter which is depending on frequency.

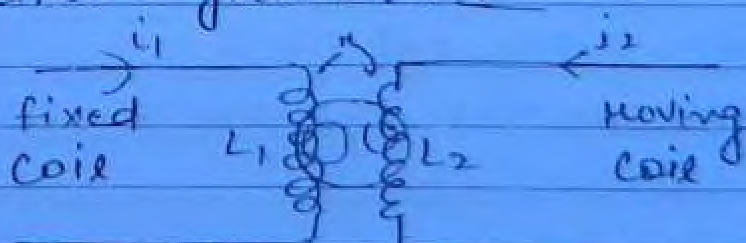
Hysteresis Error →



Nickel Iron alloy has low hysteresis area and hence it is used as a rotating Iron disc.

Eddy Current Error → At higher frequency it produces constant value of eddy current loss hence MI instrument are not used at higher frequency more than 125 Hz.

Electro dynamometer →



M = Mutual Inductance

$K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$ dM/dL

$i_1 i_2$

Date
Page

<u>Series Aiding</u> :- $L = L_1 + L_2 + 2M$

$L_1, L_2 \Rightarrow$ constants , $M \Rightarrow$ variable

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$$\boxed{\frac{dL}{d\theta} = \frac{2\,dM}{d\theta}}$$

<u>MI meter</u> :-

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$T_d \Rightarrow \frac{1}{2} I \cdot I \cdot \frac{dL}{d\theta}$$

$$T_d \Rightarrow \frac{1}{\cancel{2}} i_1 i_2 \left(\cancel{2} \frac{dM}{d\theta}\right)$$

$$\boxed{T_d = i_1 i_2 \frac{dM}{d\theta}}$$ → Electrodynamometer

$$\boxed{T_c = K\theta}$$

At balance,

$$T_c = T_d$$

$$K\theta = i_1 i_2 \cdot \frac{dM}{d\theta}$$

$$\boxed{\theta \propto i_1 i_2}$$

(1) It measures both AC and DC

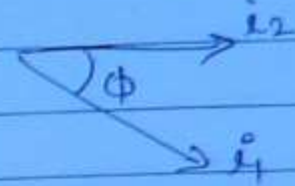
(2) In case of AC measures the RMS Quantity.

<u>Applications</u> →

(1) Ammeter

2) Voltmeter
3) wattmeter
4) powerfactor meter
5) Energymeter
6) Megger → High resistance measurement.
7) power frequency meter.

AC Measurement :-



$$i_1 = I_{m_1} \sin(\omega t - \phi)$$
$$i_2 = I_{m_2} \sin \omega t$$

$$T_{d\,avg} = \frac{1}{T} \int_0^T T_d \cdot dt = \frac{1}{T} \int_0^T \left(i_1 i_2 \frac{dM}{d\theta} \right) dt$$

for sinusoidal signal $T = 2\pi$

$$T_{d\,avg} = \frac{1}{2\pi} \int_0^{2\pi} I_{m_1} I_{m_2} \sin \omega t \cdot \sin(\omega t - \phi)\, d(\omega t) \frac{dM}{d\theta}$$

$$T_{d\,avg} = \frac{I_{m_1} I_{m_2}}{2} \cos\phi \frac{dM}{d\theta}$$

$$I_{RMS1} = \frac{I_{m1}}{\sqrt{2}} = I_1, \quad I_{RMS2} = \frac{I_{m2}}{\sqrt{2}} = I_2$$

$$T_{d\,avg} = I_1 I_2 \cos\phi \frac{dM}{d\theta}.$$

$$T_c = K\theta.$$

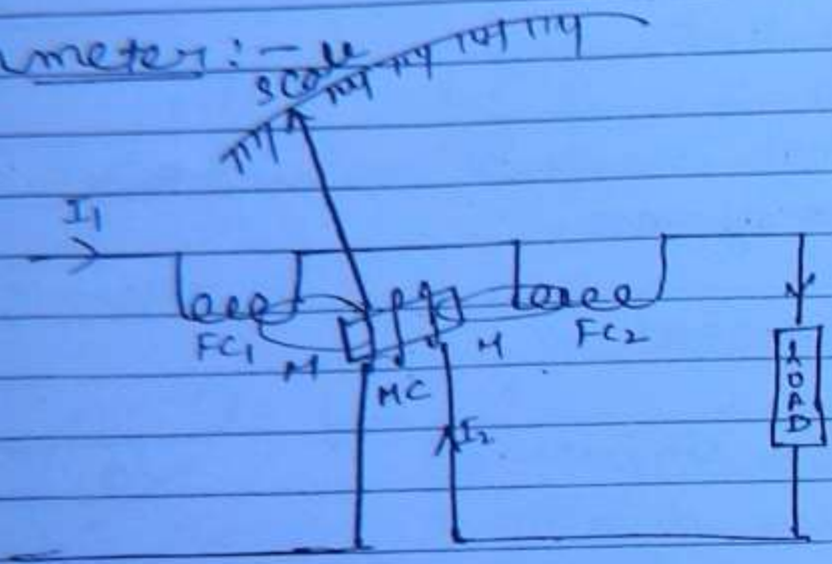
At balance $T_c = T_d$

$$K\theta = I_1 I_2 \cos\phi \cdot \frac{dM}{d\theta}$$

$$\boxed{\theta \propto I_1 I_2 \cos\phi}$$

Applications →

(1) Ammeter :-



FC_1, FC_2 → Fixed coils, MC → Moving coil

$I_1 = I_2 = I$, $\theta \propto I_1 I_2$

$\boxed{\theta \propto I^2}$ → RMS current

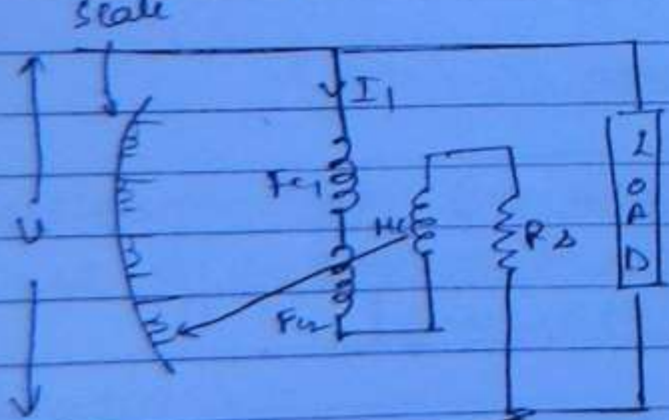
(2) Voltmeter :-

$I_1 = I_2 = V/R_s$

$\theta \propto I_1 I_2$

$\theta \propto V^2/R_s^2$

$\theta \propto V^2$ → RMS voltage.



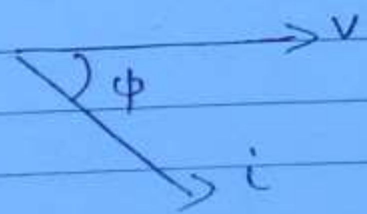
Ammeter is connected in series to the load
voltmeter is connected in parallel to the load
the voltmeter is included with series multiple resistance (Rs).

43

Power Measurement:-

$v = V_m \sin \omega t$

$i = \cancel{Em}\ I_m \sin(\omega t - \phi)$



Instantatenous power, $p = vi$

Average power, $P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} p(t) \cdot d(\omega t)$

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \sin \omega t \cdot \sin(\omega t - \phi)\, d(\omega t)$$

$$= \frac{V_m I_m}{2} \cos \phi$$

$$\Rightarrow \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi$$

$$P_{avg} = VI \cos \phi$$

$V = V_{RMS}$

$I = I_{RMS}$

ELECTRODYNAMOMETER WATTMETER:-

$$T_d = I_1 I_2 \cos \phi \cdot \frac{dM}{d\theta}$$

$I_1 = I$, $I_2 = V/R_s$

$$T_d = \frac{VI\cos\phi}{R_s} \cdot \frac{dM}{d\theta}$$

$$T_d = \frac{P_{avg}}{R_s} \cdot \frac{dM}{d\theta}$$

$$\boxed{T_d \propto P_{avg}}$$

$$T_d = T_c$$

$$\frac{P_{avg}}{R_s} \cdot \frac{dM}{d\theta} = K\theta$$

$$\boxed{\theta \propto P_{avg}}$$

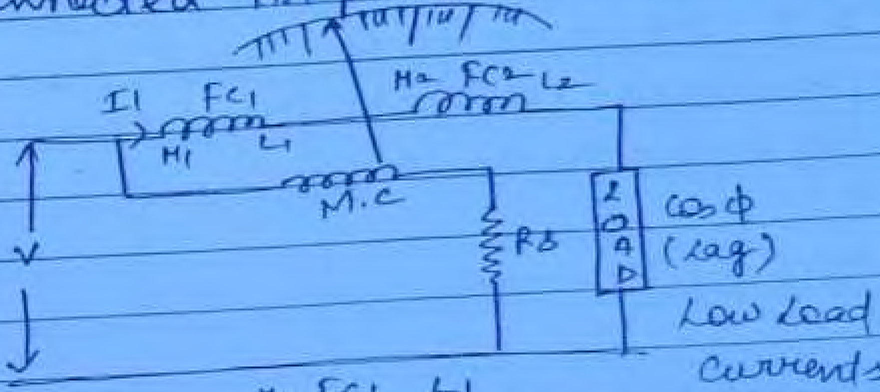
i) wattmeter measures average active power.

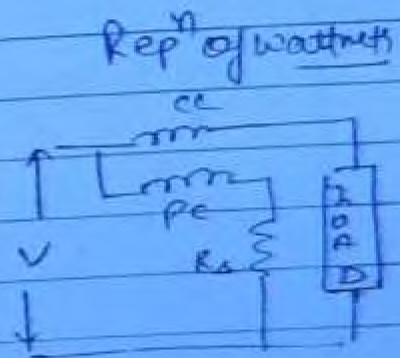
ii) Wattmeter consisting of two fixed coils F_{c1} & F_{c2} and these are connected in series to the load to measure the load current. Hence, these coils are called current coil

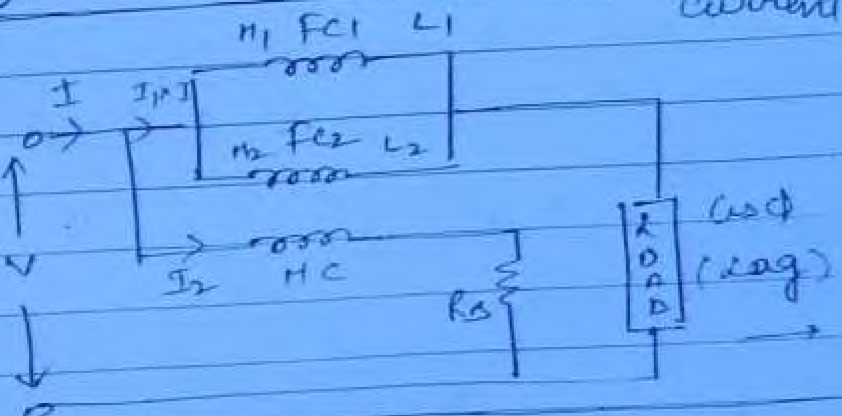
iii) Wattmeter contains moving coils which is provided with internal series multiple resistance. The moving coil is connected in parallel to the load and used to measure potential of source. Hence, moving coil is called potential coil or pressure coil.

:- A potential coil of the wattmeter is to be highly resistive so that error in measurement of power is minimum.
A potential coil of the energymeter should be highly inductive so that error in the measurement

of energy is minimum.
For small value of load current's the fixed coil's are connected in series to each other which is in-turn connected in series to the load
For large load current's the fixed coil's are connected in parallel to each other

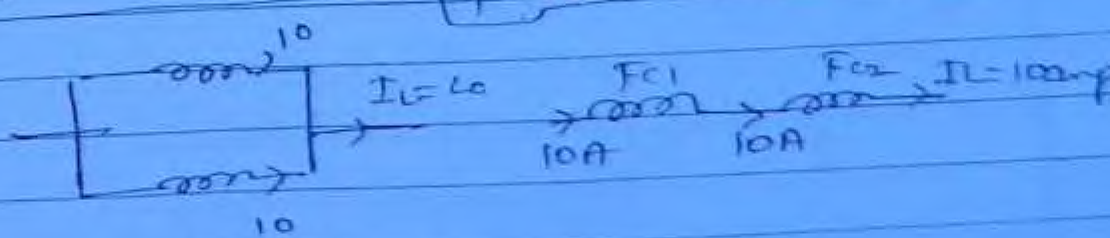






→ Large load currents

Fixed Coil (FC) - current coil (CC)
Moving coil (MC) - potential or voltage or pressure coil (PC)



Wattmeter Reading observations →

(i) For finding out the wattmeter reading observe the current flowing through the current coil which is calculated from the supply voltage and load impedance

(ii) observe the end terminals of potential coil

find the angle between potential coil current or voltage to the current coil current by using load power factor in case of 1ϕ circuit but in case of 3ϕ ckt angle is it is calculated by using phasor diagram.

If CT & PT are present in the circuit the ratio of CT & PT are to be used for calculation of present available voltage and current measured by the wattmeter.

46

Pavg calculation if Harmonic present →

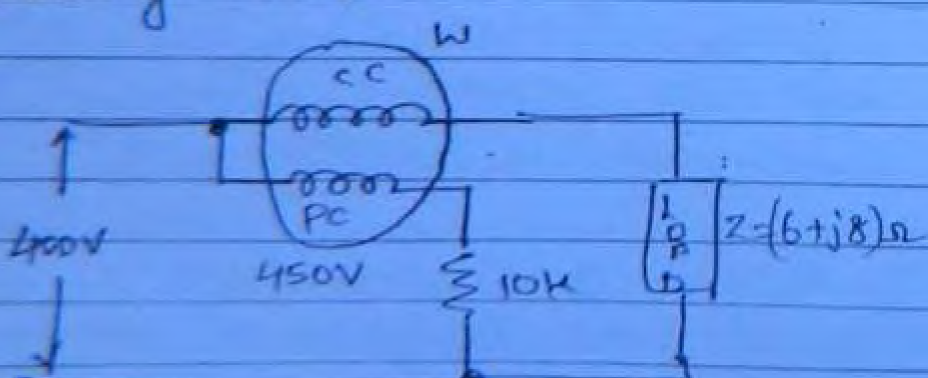
$$v = V_0 + V_1 \sin(\omega t + \phi_1) + V_2 \sin(2\omega t + \phi_2) + \cdots$$

$$i = I_0 + I_1 \sin(\omega t + \theta_1) + V_2 \sin(2\omega t + \theta_2) + \cdots$$

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} vi \, dt.$$

$$P_{avg} = V_0 I_0 + \frac{1}{2} \left[V_1 I_1 \cos(\phi_1 - \theta_1) + V_2 I_2 \cos(\phi_2 - \theta_2) + \cdots \right]$$

Find the wattmeter readings from the following ckt?



$P = VI \cos\phi$

$V = 400\ V$

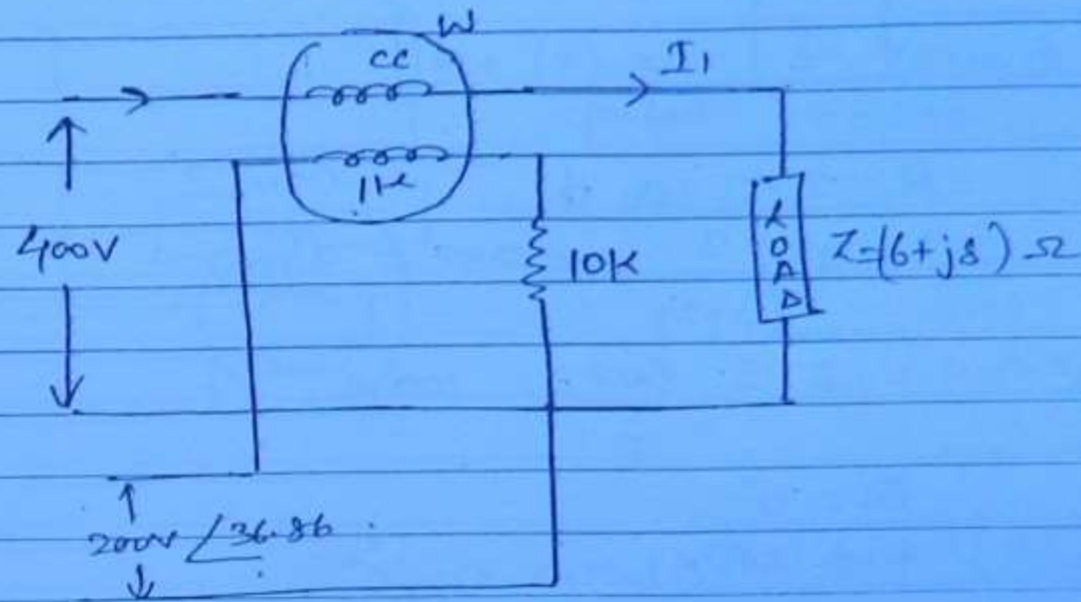
$I = \frac{400}{\sqrt{6^2+8^2}} \Rightarrow \frac{400}{10} \Rightarrow 40\ A$

(47)

$\cos\phi = 6/10 \Rightarrow 0.6$

$P = 400 \times 4\cancel{0} \times \frac{0.6}{1\cancel{0}} \Rightarrow 9600\,W \Rightarrow 9.6\,kW.$

9



$P = VI\cos\phi$

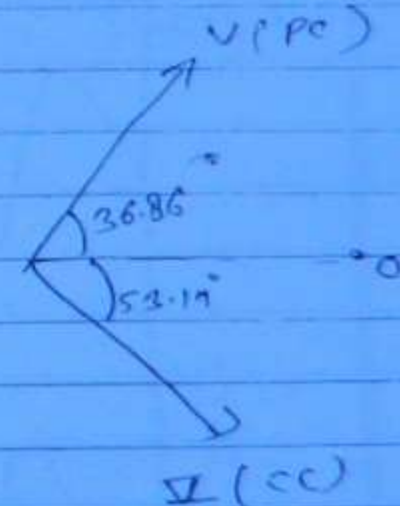
$V = 200$

$I = \frac{400}{\sqrt{6^2+8^2}} = 40\,A.$

$\cos\phi = \frac{6}{10} \Rightarrow 0.6$

$P = 200 \times 40 \times 0.6$

$= 4.8\,kW.$

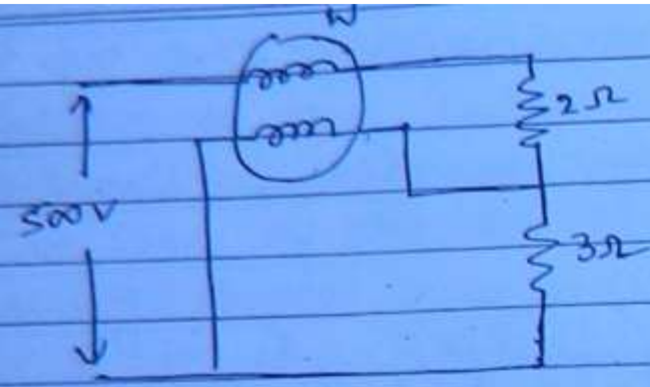


$\phi = 36.86 + 53.14^\circ \Rightarrow 90^\circ$

$P = VI\cos 90^\circ$

$\Rightarrow 0\,watt.$

Q



(48)

$P = VI\cos\phi$

$V = \frac{3}{(3+2)} \times 500 = 300V$

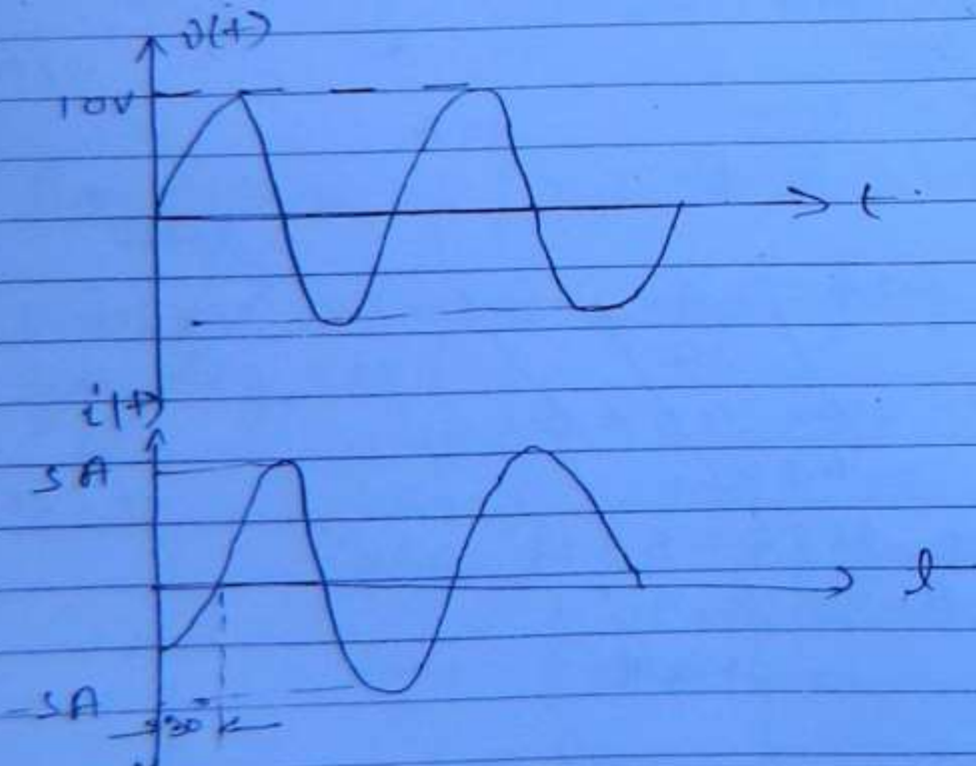
$I = \frac{500}{5} = 100A$

$\theta = 0°$

$P = 300 \times 100 \times \cos 0°$

$= 30kW$

Q



v(t) is applied to the potential coil. i(t) is applied to the current coil. Find the wattmeter reading?

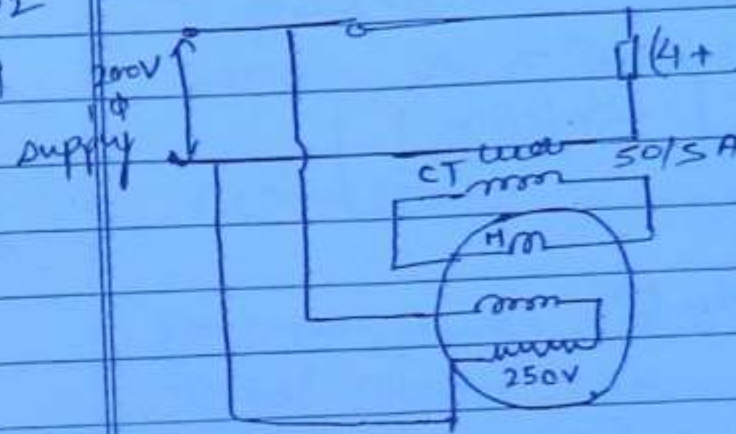
$P = VI\cos\phi$

$\Rightarrow \frac{10}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} \cos 30^\circ$

$\boxed{P = 21.65W}$

Pg42

Q21



$I_p = 40A$

$P = VI\cos\phi$

$V = 200$

$I = I_s =$

$I_p = \frac{200}{\sqrt{16+9}} \Rightarrow 40A.$

$I_s = 40 \times 5/50$

$\Rightarrow 4A.$

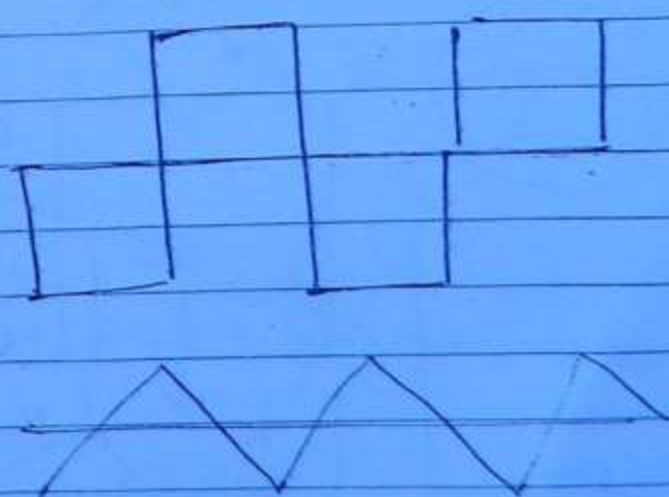
$\phi = \tan^{-1}(3/4)$

$P = VI\cos\phi$

$\Rightarrow 200 \times 4 \times \cos[\tan^{-1}(3/4)]$

$P \Rightarrow 640W.$

Q7



$v = 10 + 2\sin(2\omega t + 60^\circ) \rightarrow$ P.c

$i = 5 + 3\sin(\omega t + 30^\circ) + (1.5)\sin(2\omega t + 15^\circ)$ P.c

$$P_{avg} = 10(5) + \frac{1}{2}\left[2(0)\cos(\ ) + 2 \times 1.5\cos(60 - 15)\right]$$

$$\Rightarrow 50 + \frac{1}{2}\left[\frac{\sqrt{2}}{\cancel{2}} \times 1.5 \times \frac{1}{\sqrt{2}}\right]$$

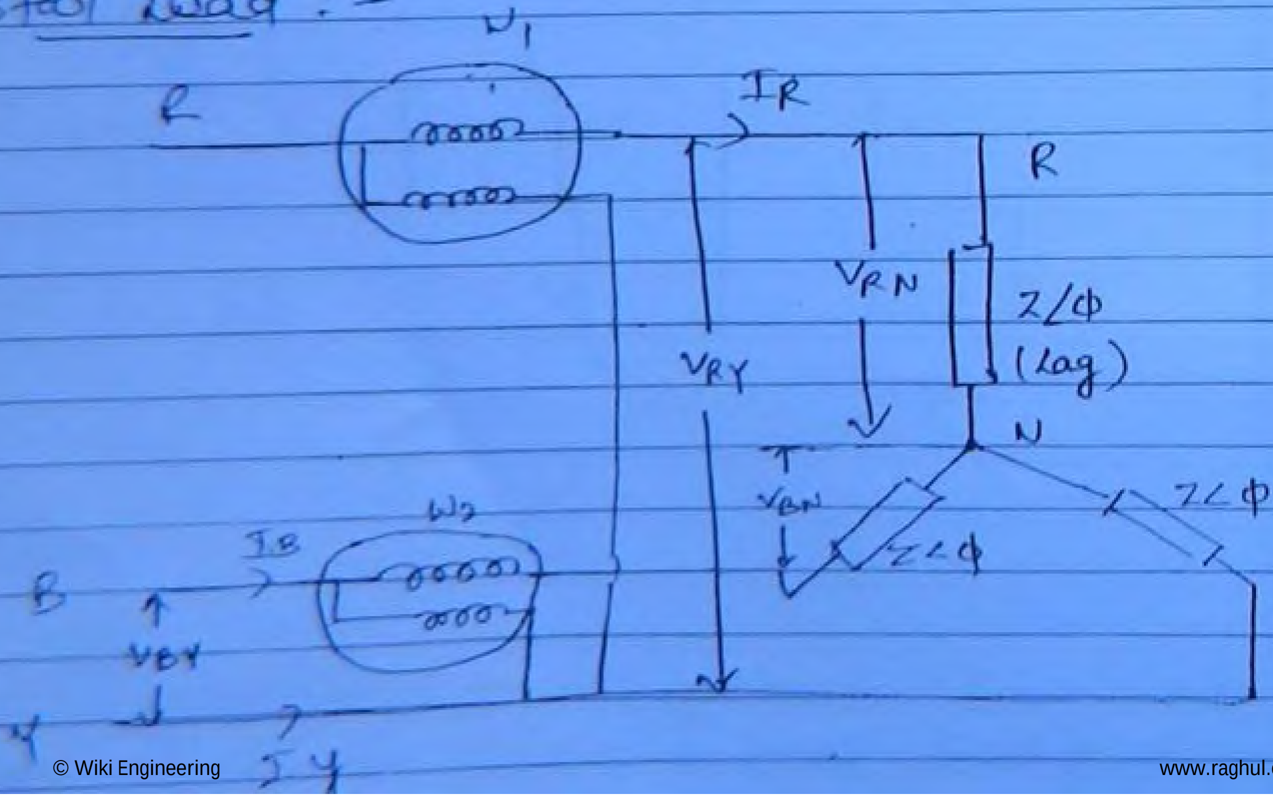
$$\Rightarrow 51.06 \text{ watt.}$$

Blondel's theorem → For measurement of power in case of n-phase balanced system. the min^m no of wattmeter required is (n-1).

For measurement of 3Φ power ~~by two~~ we require 2 wattme...

Measurement of 3 Φ ~~ba~~ power by Two Wattmeter Method :-

i) Star Load :-



For star load:-

$$V_L = \sqrt{2}\, V_{ph}, \quad I_L = I_{ph}$$

Line voltage (V_L) = V_{RY}, V_{YB}, V_{BR}.
phase voltage (V_{ph}) = V_R, V_Y, V_B.
Line or phase Current's ⇒ $\underline{I_R}$, $\underline{I_Y}$, $\underline{I_B}$.

Active power:-

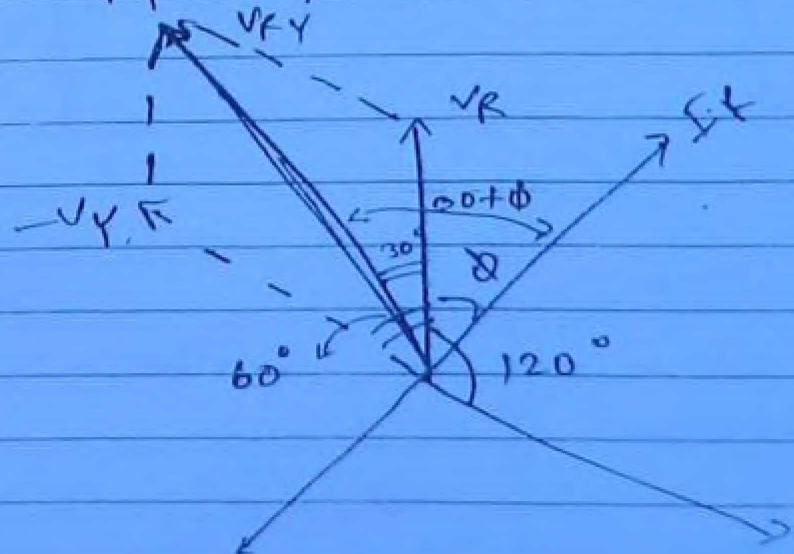
Per phase power, $P_{ph} = V_{ph} I_{ph} \cos\phi$.
Total 3phase power, $P = 3 V_{ph} I_{ph} \cos\phi$.

Reactive Power:-

Per phase ⇒ $Q_{ph} = V_{ph} I_{ph} \sin\phi$.
Total 3ϕ power ⇒ $Q = 3 V_{ph} I_{ph} \sin\phi$.

$$W_1 = V_{RY} I_R \cos \angle V_{RY} \,\&\, I_R$$

$$V_{RY} = V_R - V_Y.$$

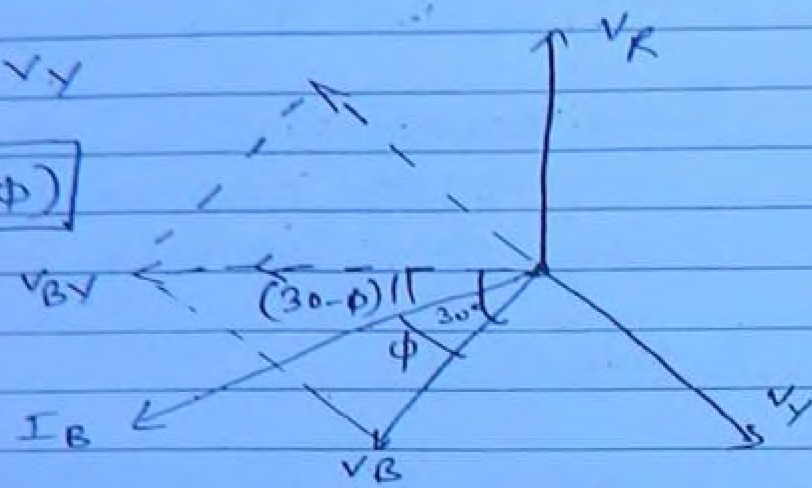


$$\boxed{W_1 = V_L I_L \cos(30+\phi)}$$

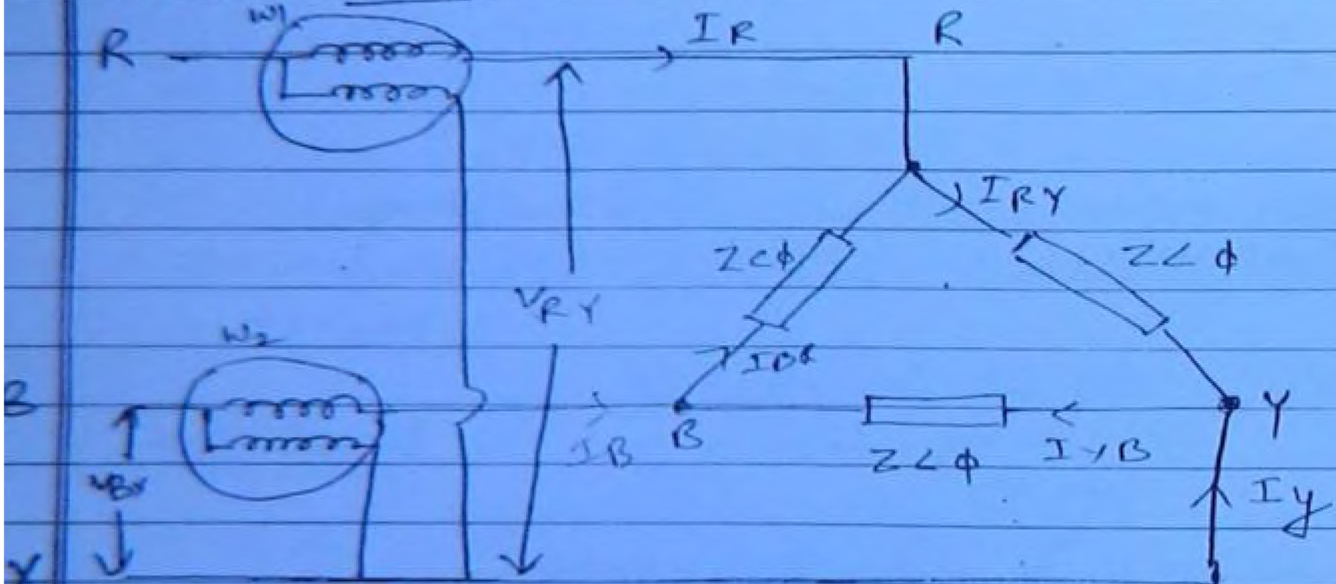
$W_2 = V_{BY} I_B \cos \angle V_{BY} \,\&\, I_B$

$V_{BY} = V_B - V_Y$

$$\boxed{W_2 = V_L I_L \cos(30-\phi)}$$



Delta Load :-



For Δ

$I_L = \sqrt{3}\, I_{ph},$

$V_L = V_{ph}$.

Line Currents $\rightarrow$

$I_L = I_R, I_Y, I_B$

Phase Current's $\rightarrow$

$I_{ph} = I_{RY}, I_{YB}, I_{BR}$

Line or phase voltages

V_{RY}, V_{YB}, V_{BR}.

Node R

$$I_R + I_{BR} = I_{RY}$$
$$I_R = I_{RY} - I_{BR}$$

Node Y

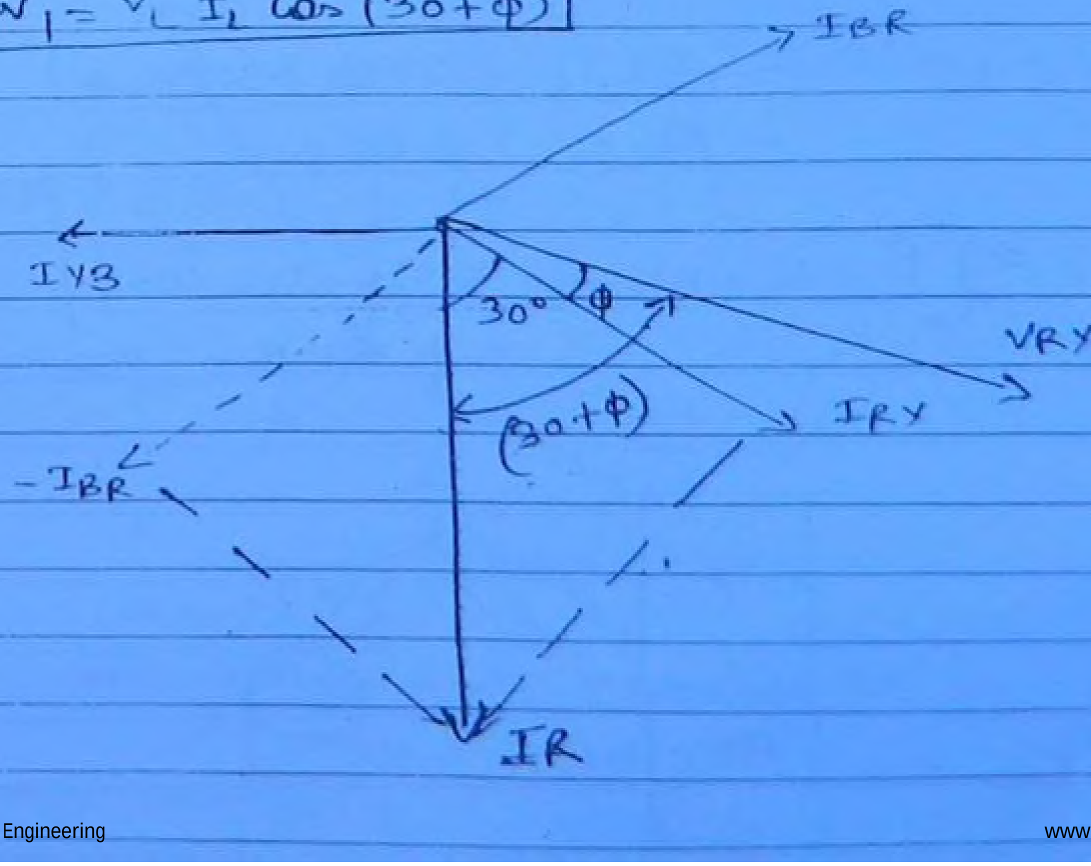
$$I_Y + I_{RY} = I_{YB}$$
$$I_Y = I_{YB} - I_{RY}$$

Node B

$$I_B + I_{YB} = I_{BR}$$
$$I_B = I_{BR} - I_{YB}$$

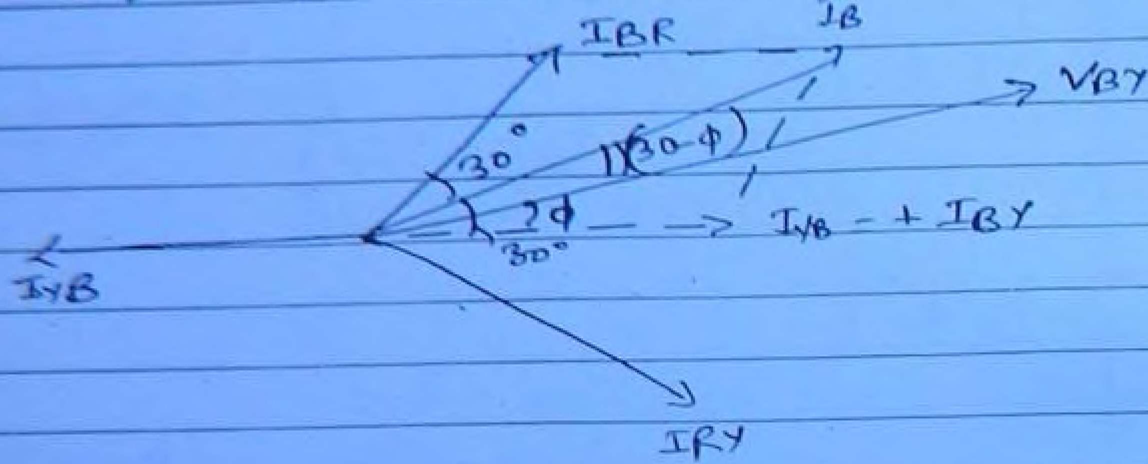
$$W_1 = \underline{V_{RY}}\, I_R \cos \angle V_{RY} \,\&\, I_R$$
$$I_R = I_{RY} - I_{BR}$$

$$\boxed{W_1 = V_L I_L \cos(30 + \phi)}$$



$$W_2 = V_{BY} I_B \cos \angle V_{BY} \& I_B$$

$$I_B = I_{BR} - I_{YB}$$



$$\cancel{W_2 = V_{BY} I_B \cos \angle V_{BY}}$$

$$W_2 = V_L I_L \cos(30 - \phi)$$

Active Power (P) :-

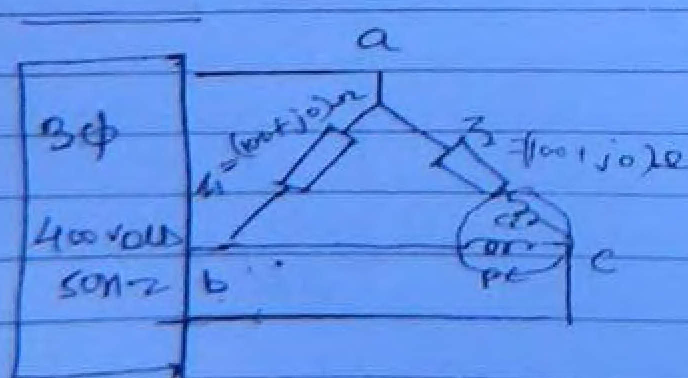
$$W_1 + W_2 = V_L I_L \left[\cos(30+\phi) + \cos(30-\phi)\right]$$

$$= V_L I_L \left[2 \cos 30 \cdot \cos\phi\right]$$

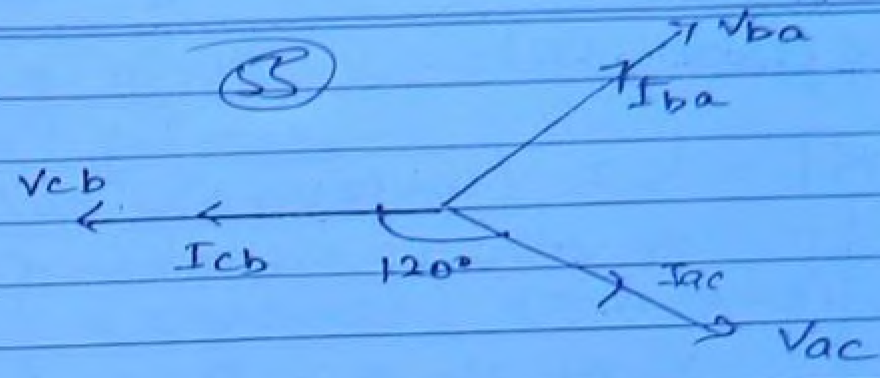
$$= \sqrt{3}\, V_L I_L \cos\phi$$

$$\therefore \left[P = W_1 + W_2 = 3 V_{ph} I_{ph} \cos\phi\right]$$

$$= 3\phi \text{ Active power.}$$

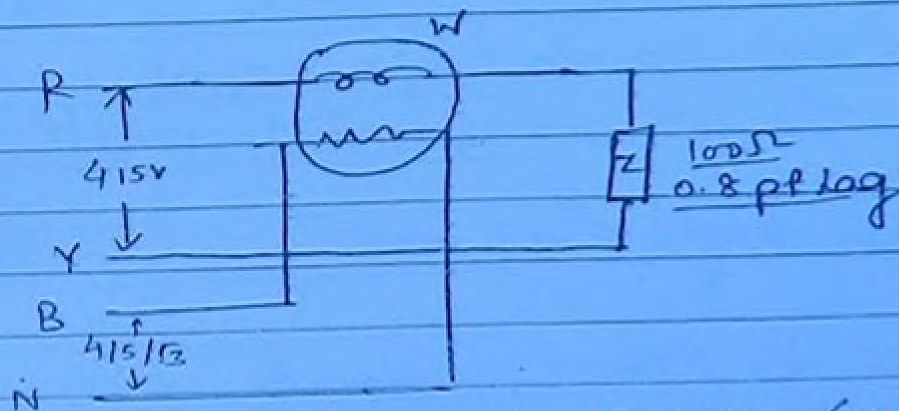


$$W_1 = V_{cb} I_{ac} \cos \angle V_{cb} \& I_{ac}$$



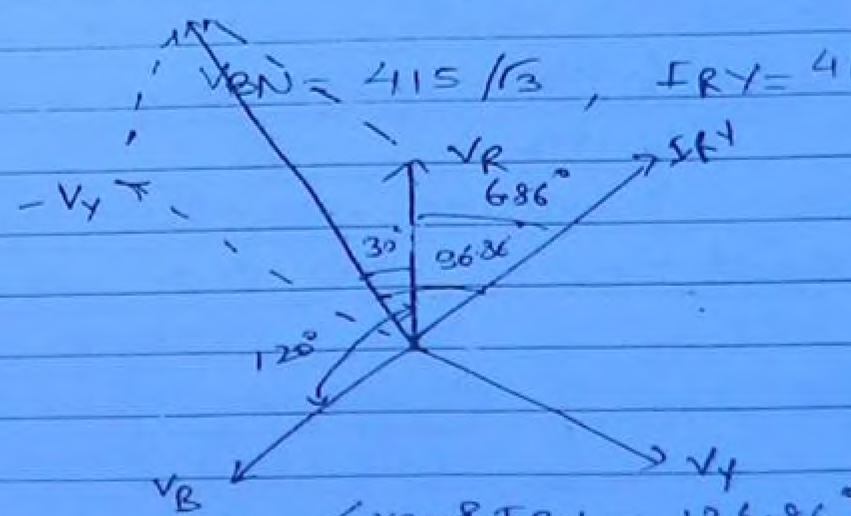
$$W = 400 \times \frac{400}{100} \times \cos 120^\circ$$

$$\Rightarrow 800\,W.$$



$$W = V_{BN} I_{RY} \cos \angle V_{BN} \& I_{RY}$$

$V_{BN} = 415/\sqrt{3}$, $I_{RY} = 415/100 = 4.15A$.



$$\angle V_{BN} \& I_{RY} = 126.86^\circ$$

$$W = 415/\sqrt{3} \times 4.15 \times \cos(126.86^\circ)$$

$$\Rightarrow -597\,W$$

Reactive power (Q):-

$$|W_1 - W_2| = V_L I_L [\cos(30+\phi) - \cos(30-\phi)]$$
$$\Rightarrow V_L I_L (2 \sin 30 \sin \phi)$$

$$|W_1 - W_2| = V_L I_L \sin \phi = \sqrt{3} V_{ph} I_{ph} \sin \phi$$
$$\Rightarrow \sqrt{3} Q_{ph}.$$

$$Q_{ph} = \frac{|W_1 - W_2|}{\sqrt{3}}$$

3ϕ Reactive power $Q = 3 Q_{ph}$

$$\boxed{Q = \sqrt{3}(W_1 - W_2)}$$

Power factor Angle (ϕ) :-

$$W_1 - W_2 = \sqrt{3} V_{ph} I_{ph} \sin \phi \quad - (1)$$
$$W_1 + W_2 = 3 V_{ph} I_{ph} \cos \phi \quad - (2)$$

$$\frac{(1)}{(2)} = \frac{W_1 - W_2}{W_1 + W_2} = \frac{\tan \phi}{\sqrt{3}}$$

for Lag Load :-

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

for lead Load :-

$$\phi = -\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$$

Reading of W_1, W_2 for different Load power factors:-

ϕ	$\cos\phi$	W_1 $V_L I_L \cos(30+\phi)$	W_2 $V_L I_L \cos(30-\phi)$	W W_1+W_2	observatio
$0°$	1	$\sqrt{3}/2\, V_L I_L$	$\sqrt{3}/2\, V_L I_L$	$\sqrt{3}\, V_L I_L$	Both watt shown so $\cos\phi=1$ $W_1=W_2$
$30°$	$0.866°$	$V_L I_L/2$	$V_L I_L$	$3/2\, V_L I_L$	$W_2=2W_1$
$60°$	0.5	0	$\sqrt{3}/2\, V_L I_L$	$5/2\, V_L I_L$	$W_1=0$ $W_2=W$
$90°$	0	$-V_L I_L/2$	$V_L I_L/2$	0	$W_1=-ve$ $W_2=+ve$

ote:- If one of the watlmeter, shows negative value, reverse either CC or PC terminals and Record with negative sign.

→ In two wattmeter method If the load p.f is between 0 to 0.5 than one of the wattmeter show negative value bcoz the phase angle b/w voltage & current is more than $90°$ ($\cos(90+\phi) = -ve$).

Q) In the measurement of power on balanced load by two wattmeter method in a 3ϕ ckt reading's to the wattmeter are 3kw & 1kw respectively the late being ~~app~~ obtain after reversing the connection to the current coil. The P.f of the load is

a) 0.277 b) 0.277' c) 0.554 d) .966

(58)

$W_1 = 3 kW$

$W_2 = -1 kW.$

$$\phi = \tan^{-1} \sqrt{3} \left(\frac{3-(-1)}{3+(-1)} \right)$$

$$\Rightarrow \tan^{-1} \sqrt{3} \left(\frac{\cancel{4}^{2}}{\cancel{2}_{1}} \right)$$

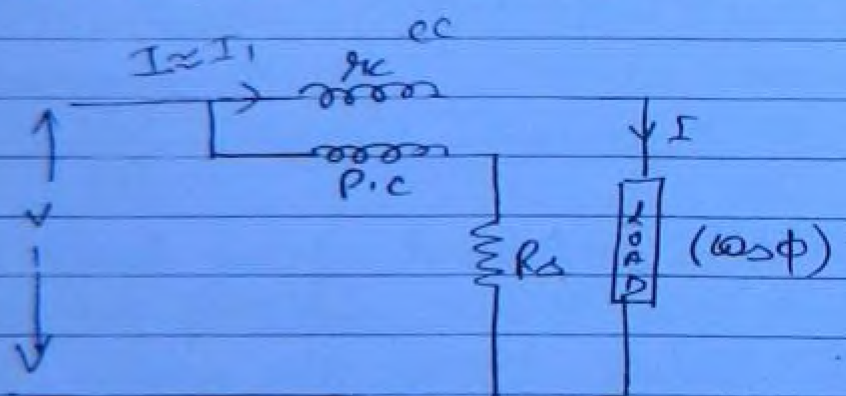
$$\Rightarrow \tan^{-1} (2\sqrt{3})$$

$$\boxed{\cos \phi = 0.277}$$

Errors in wattmeter →

Error due to Connection :-

potential coil is on source side:-



Rs is large hence I_2 is negliable

r_C = C.C internal resistance.

Measured power P_m = True load power (P_T) + power loss in C.C resistance(r_c)

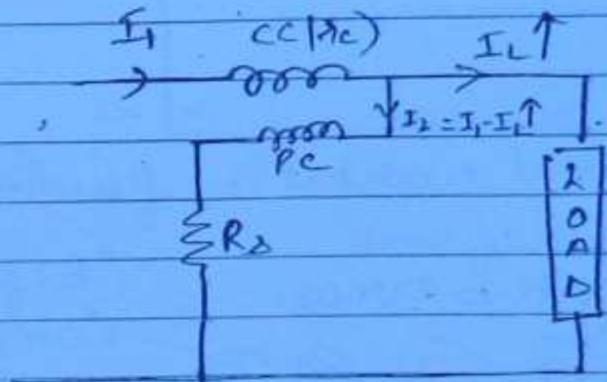
$$\boxed{P_m = P_T + I^2 r_C}$$

$$\% \text{ Error} = \frac{P_m - P_T}{P} \times 100.$$

$$\boxed{\% E_r = \frac{I^2 r_c}{P_T} \times 100}$$

Note:- Error if load if I is small. Hence this is used for low load current's. (oc's test)

b) P.C on Load side:-



power loss in r_c is very small compared to R_s loss.

$P_m = P_T$ + power loss in PC Resistance (R_s)

$P_m = P_T + I_2^2 R_s$; $I^2 R_s = V^2/R_s$

$$\% \text{ Error} = E_r = \frac{P_m - P_T}{P_T} \Rightarrow \frac{I_2^2 R_s}{P_T} \times 100$$

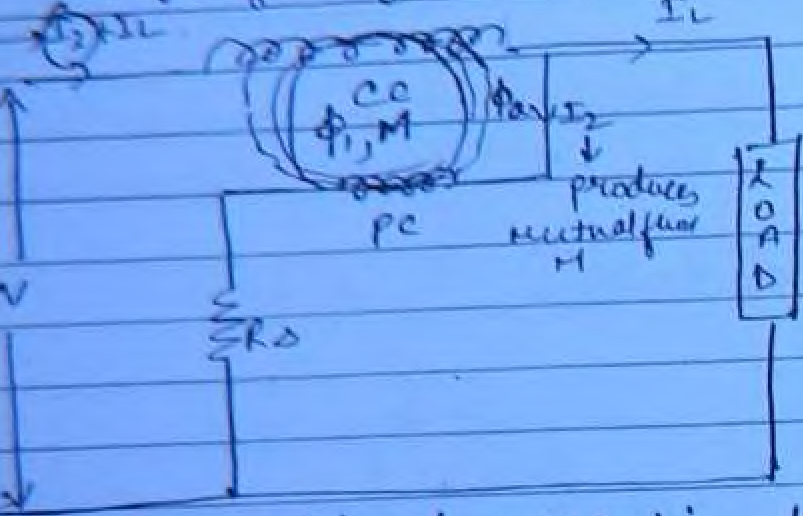
Note:- E_r is low if I_2 is small. ($I_2 = I_1 - I_L$). If I_L is large I_2 is small. Hence used for Load currents. [SC test's].

Error's are equal.

$$\boxed{I^2 r_c = \frac{V^2}{R_s}}$$

2) Error due to P.C current flowing in CC:-

only I_L flowing producing flux ϕ_1

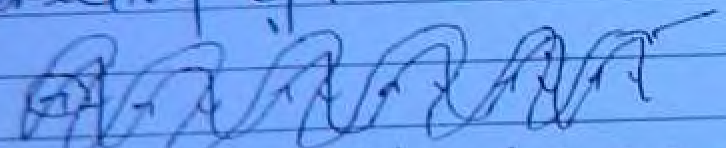


Extra current I_2 belongs to PC which is flowing in CC it produces ϕ_a that EMF is induced in coil.

Due to this additional current is flowing. This causes ~~losses~~ error.

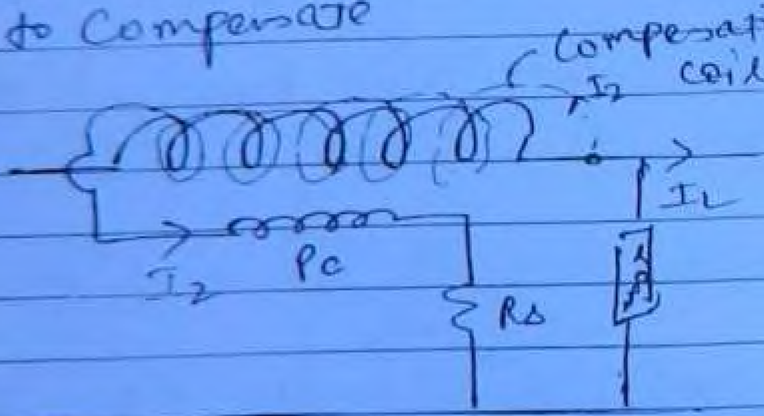
for compensating of this error.

(Bifilar winding) Compensating coil.



CC carries a additional current $I_2 \rightarrow \phi a$.

ϕa to Compensate

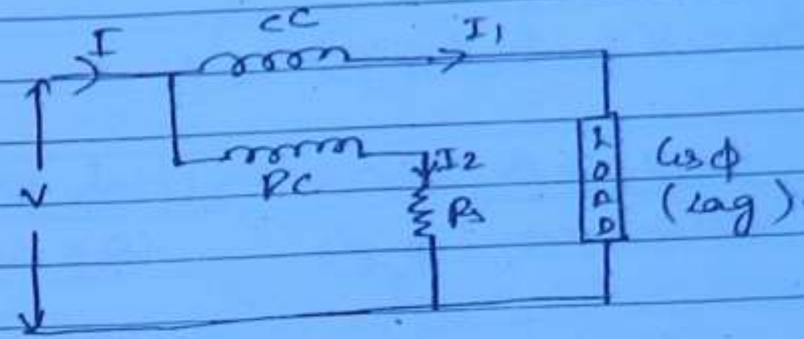


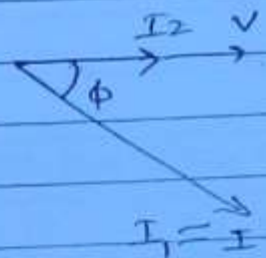
To compensate current in the potential coil.

→ Compensating coil is connected in series to the potential coil to reduce the power loss in the potential coil due to potential coil current flowing in the current coil.

3) Error due to potential Coil Inductance (L_P):-

without Inductance in PC :- [Highly Resistive]

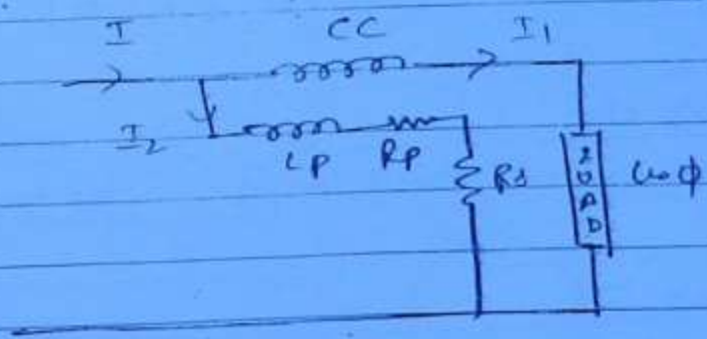




$$T_d = \frac{VI}{R_s} \cos\phi \cdot \frac{dM}{d\theta}$$

True power $P_T = VI\cos\phi$

with Inductance of P.C:-



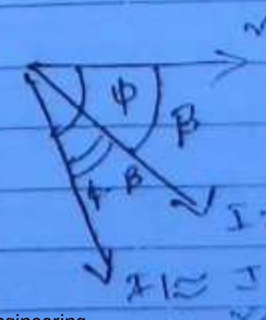
Inductance of P.C = L_P.

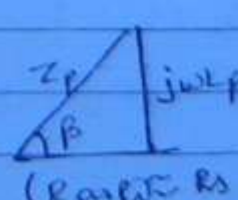
Resistance of P.C = R_P ($R_P << R_s$)

series resistance = R_s

Impedance Angle of P.C => $\beta = \tan^{-1}\left(\frac{\omega L_P}{R_s}\right)$

$$T_d \propto I_1 I_2 \cos(\phi - \beta) \frac{dM}{d\theta}$$





$$\cos\beta = \frac{R_s}{Z_s}$$

$I_1 \approx I$

$I_2 \approx V/Z_P = \frac{V\cos\beta}{R_s}$

$(R_P + R_s) = R_s$

$$\left[\frac{1}{Z_P} = \frac{\cos\beta}{R_s}\right]$$

$$T_d \propto \frac{VI\cos(\phi-\beta)\cos\beta}{R_L}\left(\frac{dM}{d\theta}\right)$$ (62)

measured power → $$P_m = VI\cos(\phi-\beta).\cos\beta$$

$$\frac{P_T}{P_m} = \frac{VI\cos\phi}{VI\cos\beta\cos(\phi-\beta)} = \frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)}$$

$$\boxed{P_T = P_m.\frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)}}$$

$$\text{Connection factor} = \frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)}$$

$$\frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)} = \frac{\cos\phi}{\cos\beta\,[\cos\phi\cos\beta+\sin\phi\sin\beta]}$$

$$\Rightarrow \frac{\cos\phi}{\cos^2\beta\cos\phi\,[1+\tan\phi\tan\beta]} \Rightarrow \frac{\sec^2\beta}{(1+\tan\phi\tan\beta)}$$

$$\sec^2\beta = 1+\tan^2\beta$$

for small β, $\tan^2\beta \approx 0$, $1+\tan^2\beta \approx 1$

$$\frac{P_T}{P_m} = \frac{\cos\phi}{\cos\beta.\cos(\phi-\beta)} \approx \frac{1}{1+\tan\phi\tan\beta}$$

$$\%\text{ Error} = \frac{P_m-P_T}{P_T}\times 100 = \tan\phi\tan\beta\times 100.$$

Error:- $$P_m - P_T = P_T.\tan\phi\tan\beta$$
$$= VI\cos\phi.\frac{\sin\phi}{\cos\phi}.\tan\beta$$

$$P_m - P_T = VI\sin\phi\tan\beta.$$

(1) For Lag Load:- $\phi = +ve$

$$P_m = P_T + VI\sin\phi\tan\beta$$

$$\boxed{P_m > P_T}$$

(2) For Lading Load:- $\phi = -ve$

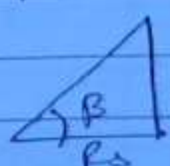
$$P_m = P_T - VI\sin\phi\tan\beta$$

$$\boxed{P_m < P_T}$$

Error for different load power factors:- [UPF watt meter]

unity power factor.

ϕ	$\cos\phi$	$\sin\phi$	$P_m - P_T = VI\sin\phi\tan\beta$	observation
0°	1	0	0	Min Error
30°	.866	0.5	$0.5\,VI\tan\beta$	
60°	0.5	0.866	$0.866\,VI\tan\beta$	
90°	0	1	$VI\tan\beta$	Max Error

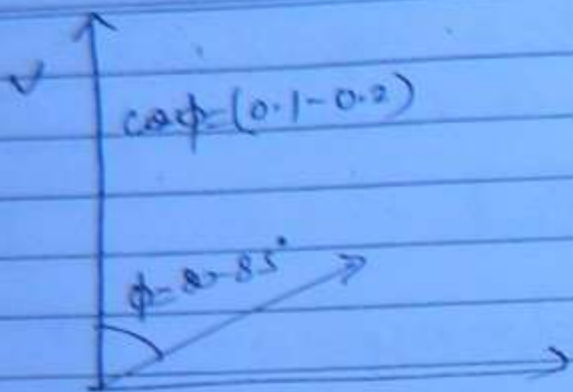


$$\text{Error} = P_m - P_T = VI\sin\phi\tan\beta$$

$$\tan\beta = 0 \Rightarrow \beta = 0 \Rightarrow \text{Error} = 0.$$

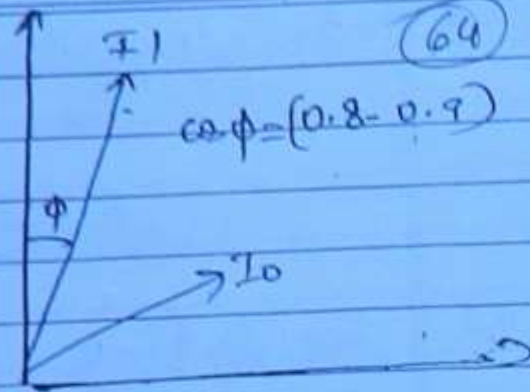
The error is becoming zero if β = zero i.e L_p should be very low compare to R_s hence potential coil should also be highly resistive for measuring power more accurately

oc test



(LPF)

Page
S.C test



(UPF)

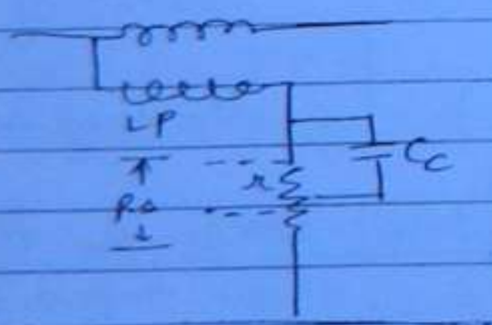
LPF wattmeter :-

(1) $\downarrow T_d = \frac{VI\cos\phi\downarrow}{R_s \downarrow} \frac{dM}{d\theta}$

(1) Low power factor :- $\cos\phi \downarrow$, $T_d \downarrow$

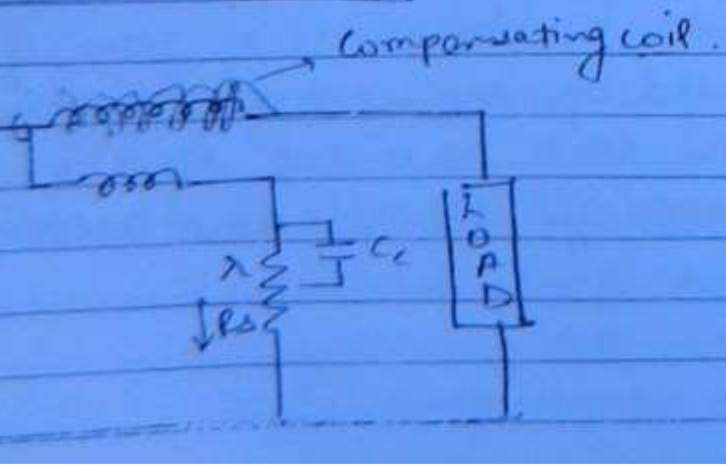
$R_s \downarrow$, $T_d \uparrow$

(2) $\left.\begin{matrix} P_m - P_T \\ \text{error} \end{matrix}\right\} = VI\sin\phi \tan\beta$ $\quad \tan\beta = \frac{\omega L_p \downarrow}{R_s}$



$\boxed{C_c = L_p / r^2}$

(3)



LPF wattmeter's are designed to measure the power more accurately in case of low power factor load condition by compensating the effect of potential coil inductance

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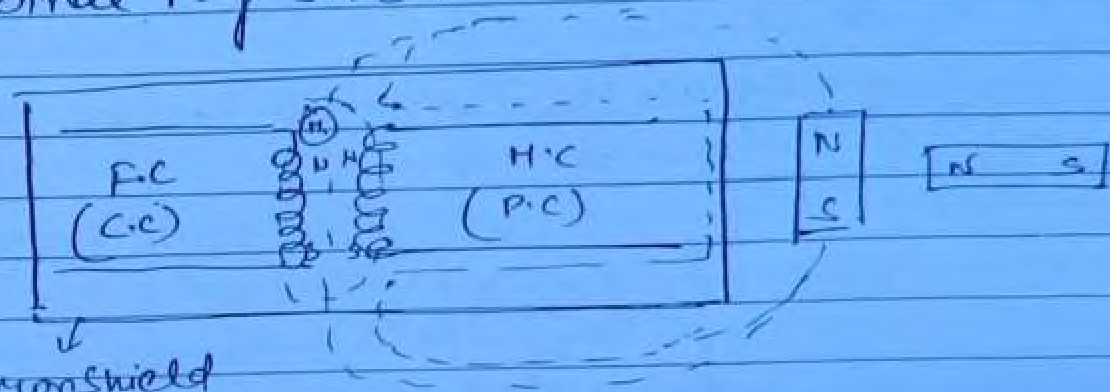
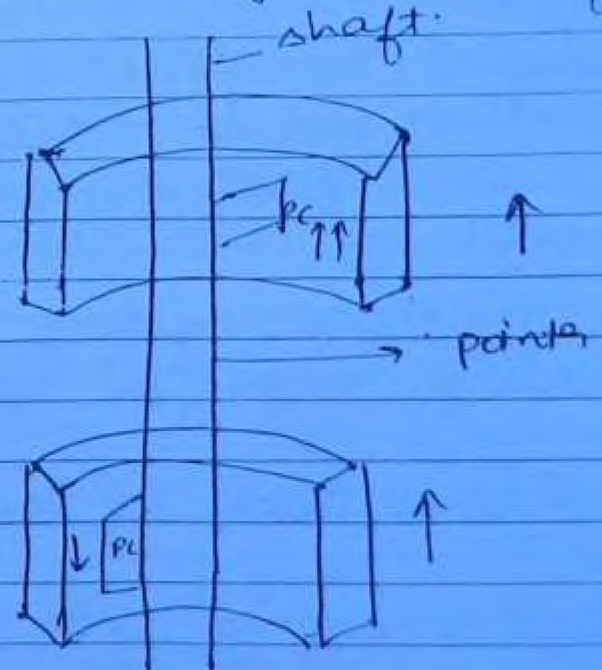
(4) External Mag Fields →



Lab Meters.

Astatic System :- High Precision (or) More Accurate wattmeter



In Laboratory instrument are iron shielded to reduce the effect of external magnetic field. In high accurate meters astatic system is used which the two potential coil's are kept in

the Opposite side of spindle (shaft): In the external magnetic field is applied than it is opposed by one potential coil and added by the other potential coil the net effect on the shaft will be zero so that external magnetic field effect is nullified

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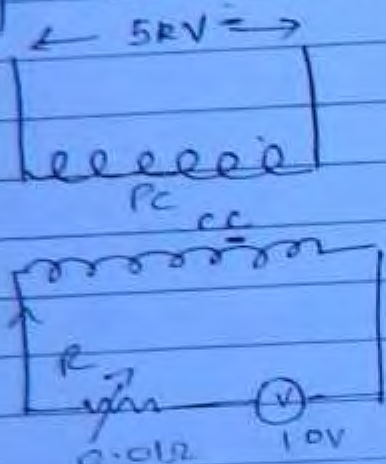
→ Testing of wattmeter and Energy Meter →



$\frac{5\times10^6}{5\times10^3} = 1000A$

Energy ⇒ $5\times10^3 \times 1000 \times 2$ hr
⇒ 10,000 kwh
⇒ 10,000 × Rs 5
⇒ Rs 50,000/-

$\frac{10}{.01} = I$
$= 1000$ A

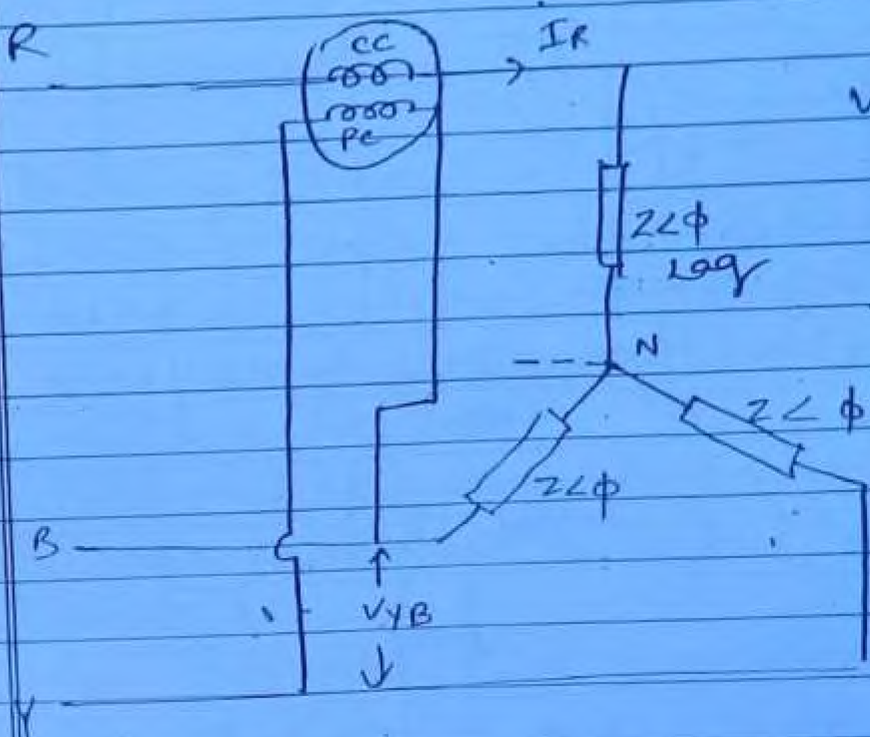
Energy = 1000×10×2 ⇒ 20,000
⇒ 20kwh = 20 units
20×5 = Rs 100/-

Phantom or fictitious Loading is used for testing of energy meter and watt

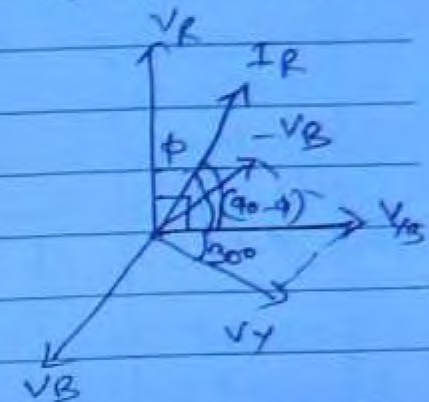
during Calibration work of these meter's is minimized.
Potential Coil is excited with rated voltage and the current coil is applied with small dc potential with a variable resistor by adjusting the variable resistor rated current is passing through the current coil so that energy consumed by the load can be recorded

by the meter.

Measurement of Reactive power by wattmeter →



$W = V_{YB} I_R \angle V_{YB} \& I_R$



In the wattmeter current coil is connected to one of the phase (R phase) of a 3ϕ system and the potential coil is connected b/w the remaining two phases (Y and B) then the wattmeter measures reactive power.

$$W = \sqrt{3}\, V_{ph} I_{ph} \sin\phi$$

$$W = \sqrt{3}\, Q_{ph}$$

$$\boxed{Q = 3 \cdot Q_{ph} = \sqrt{3}\, W}$$

Ch-4

Q1

$$W = V_{ph} I_{ph} \cos\phi$$

$$400 = V_{ph} I_{ph} (0.8)$$

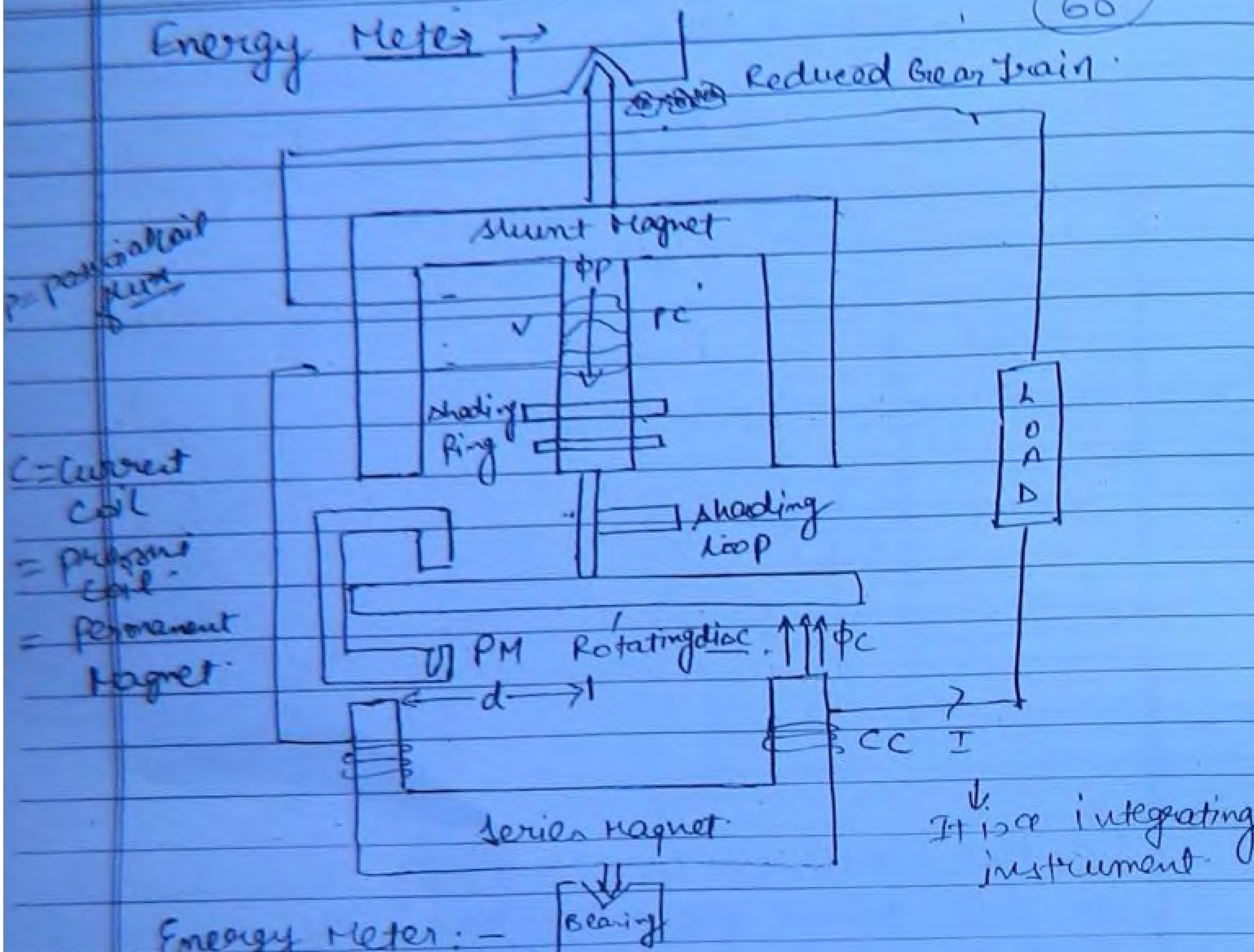
$$V_{ph} I_{ph} = 500$$

$$W = \sqrt{3}\, V_{ph} I_{ph} \sin\phi$$

$$\Rightarrow \sqrt{3} \times 500 \times 0.6$$

$$\Rightarrow 519.6 \text{VAR}$$

Energy Meter →



Energy Meter :-

Energy Meter = Power × Time

$= P \times t$

$= VI\cos\phi \times t$

$\boxed{1\ \text{Kwhr} = 1\ \text{unit}}$

Mechanisms :-

1. Driving Mechanism — CC, PC
2. Rotating Mechanism — Al Disc
3. Braking Mechanism — permanent magnet
4. Recording (or) Registering Mechanism — Reduced Gear Train.

Energy Meter working on a principle of Induct used to measure energy consumed by the load and this energy is cummulatively added over a period of time. Hence it is an integrating instrument.

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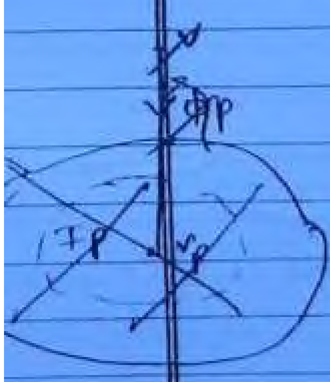
(i) Driving Mechanism →

→ Driving Torque (Td):– Energy is proportional to power ($VI\cos\phi$). For Measurement of the power the voltage across the load is measured by the potential coil.

→ potential coil should be highly inductive so that error is minimum in the measurement of energy

→ The current coil is connected in series to the load which is used to measure current flowing through the load.

→ The potential coil produce a flux (ϕ_p) which is flowing through the rotating disc induces an eddy voltage (e/V_p) which is lagging ϕ_p by 90°. This V_p produces an eddy current I_p which lags V_p by angle (α) (impedance angle ϕ of rotating disc).

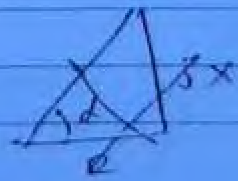


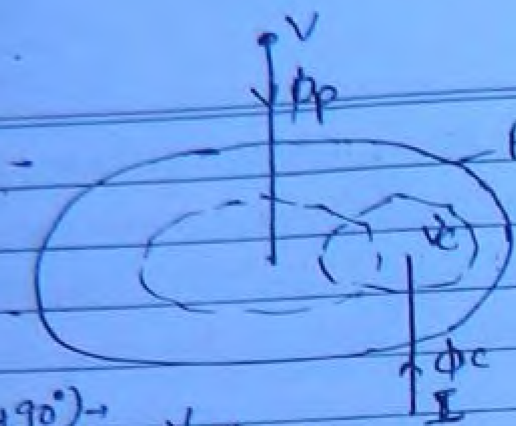
$$\phi_p = \phi_m \sin\omega t.$$

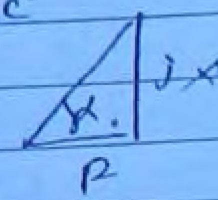
$$V_p = -\frac{d\phi_p}{dt} \Rightarrow \phi_m \omega \cos\omega t.$$

$$V_p = \phi_m \omega \sin(\omega t - 90°)$$

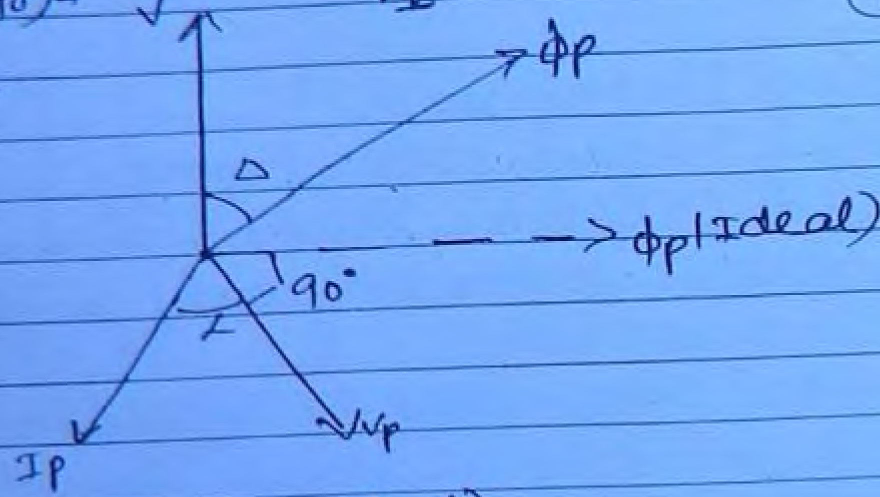
V_p lags ϕ_p by 90°.





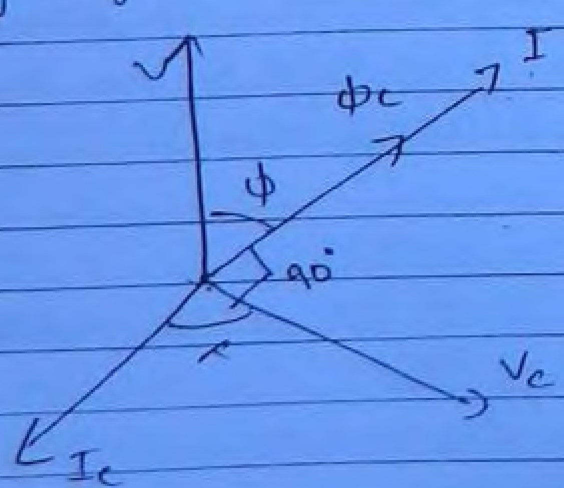


(Vp leads ϕ_p by 90°)→



Driving Torque :- (T_d).

The current coil current ~~flux~~ produces flux ϕ_c which is passing through the rotating disc induces an eddy voltage (V_c) lagging ϕ_c by 90°. Eddy current (I_c) produced due to V_c which lags V_c by angle α.



The interaction of ϕ_p and i_c produces a driving torque

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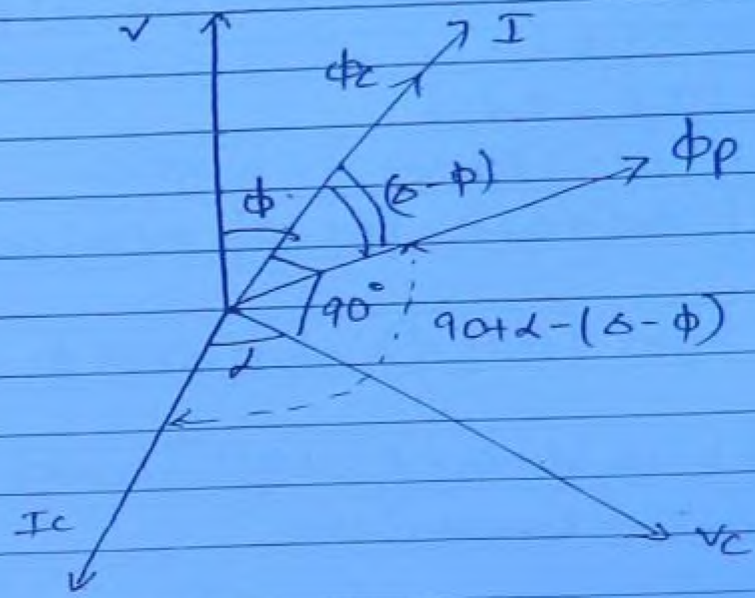
$$T_{d1} \propto \phi_p I_c \cos \angle \phi_p \& I_c$$



$\phi_p \propto V$

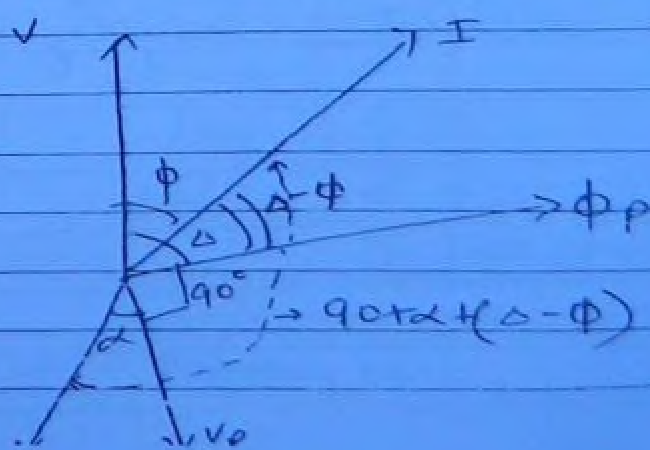
$I_c \propto V_c \propto \phi_c \propto I$

$\phi_p I_c \propto VI$

$$T_{d1} \propto VI \cos[(90+\alpha)-(\Delta-\phi)]$$

Interaction of ϕ_c and I_p produces driving torque T_{d2} producing ϕ_c and I_p.

$$T_{d2} \propto \phi_c I_p \cos \angle \phi_c \& I_p$$



$\phi_c \propto I$

$I_p \propto V_p \propto \phi_p \propto V$

$\phi_c I_p \propto VI$

$$\boxed{T_{d_2} \propto VI \cos[90 + \alpha + (\Delta - \phi)]}$$

$T_d \propto |T_{d_1} - T_{d_2}|$

$\propto VI [[\cos(90 + \alpha - (\Delta - \phi))] - \cos(90 + \alpha + \Delta - \phi)]$

$\propto VI [2 \sin(90 + \alpha) \sin(\Delta - \phi)]$

$\propto VI [2 \cos\alpha \sin(\Delta - \phi)]$

$2\cos\alpha = \text{constant}$

$T_d \propto VI \sin(\Delta - \phi) = P_m =$ Measured power.

True power $P_T = VI\cos\phi$

Error $= P_m - P_T$

Error $= VI[\sin(\Delta - \phi) - \cos\phi]$

$\%\ \text{Error} = \frac{P_m - P_T}{P_T} \times 100$

$\%\ E_r = \frac{\sin(\Delta - \phi) - \cos\phi}{\cos\phi} \times 100$

If $\Delta = 90°$, then the Error is zero so that potential coil should be highly Inductive is to be used.

Q2. $V = 220V$
$I = 5A$
$\Delta = 85^\circ$.

(1) $\cos\phi = UPF \Rightarrow \phi = 0^\circ$

Error $= VI\,[\sin(\Delta - \phi) - \cos\phi]$

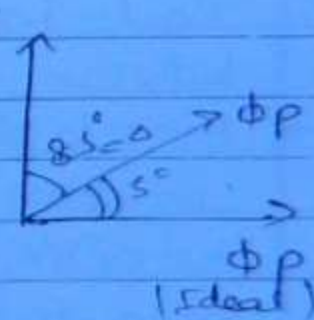
$\Rightarrow 220 \times 5\,[\sin(85 - 0^\circ) - \cos 0^\circ]$

$\Rightarrow -4.2W$

(2) $\cos\phi = 0.5, \quad \phi = 60^\circ$

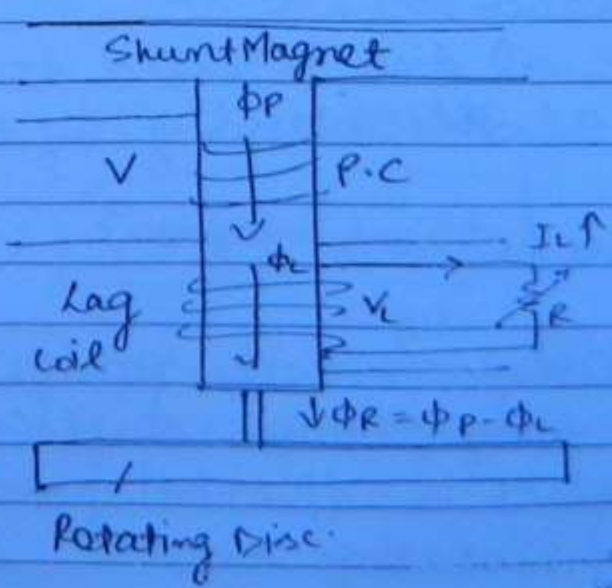
Error $= 220 \times 5\,[\sin(85 - 60) - \cos 60^\circ]$

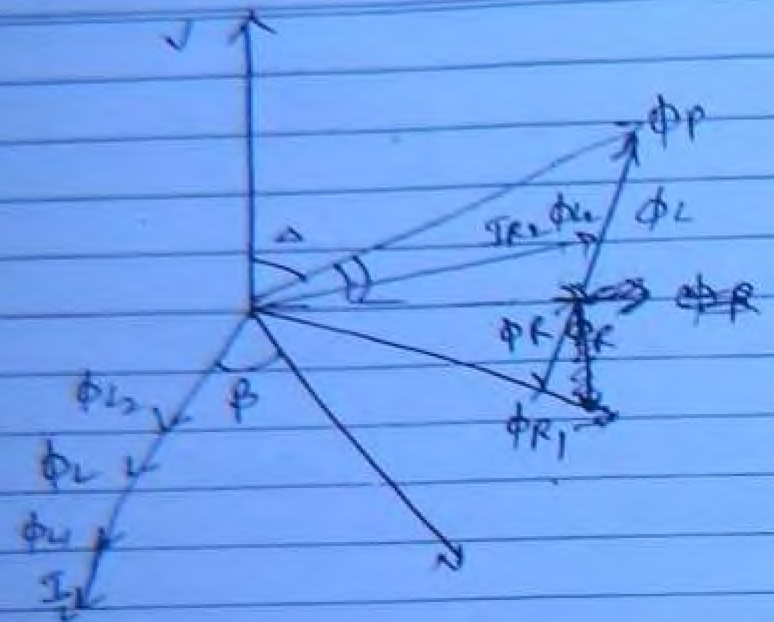
$\Rightarrow -85.1W$



→ As the power factor of the load is decreasing than error ↑ses if the potential coil is not inductive.

Lag Coil / Quadrature / power factor / shading Ring compensation :-





→ lagging coil or shading ring are used to improve the powerfactor of the potential coil in order to maintain 90° phase disp between ϕ_R & V.

→ A variable resistor is used for controlling ϕ_L in case of lag coil.

→ The position of the shading ring is adjusted for controlling ϕ_L in case of shading ring.

Shading ring is made of copper.

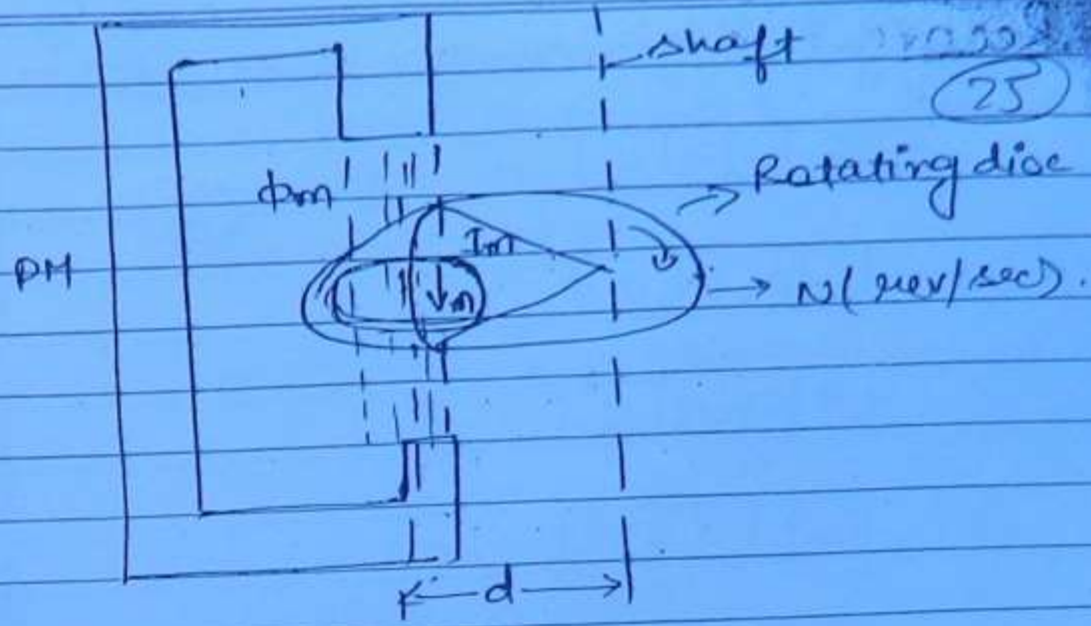
~~Braking~~ Torque :- (T_D)

Rotating Mechanism :

Low weight al disc is used to reduce the friction at the bearing of the shaft or spindle.

Breaking Mechanism →

1) Breaking Torque (T_B) :- The position of the permanent magnet wrt shaft (centre of the rotating disc) is adjusted for controlling the speed of the rotating disc.



$T_B \propto \phi_m I_m d$

$I_m \rightarrow V_m \Rightarrow \dfrac{\phi_m}{t} = N\phi_m$

$T_B \propto \phi_m N \phi_m \cdot d$

$T_B = K \cdot \phi_m^2 N \cdot d \qquad N = \dfrac{T_B}{K \phi_m d}$

$\phi_m, d, K =$ constant's. $\boxed{N \propto 1/d}$

$\boxed{T_B \propto N}$

At Const speed :-

$T_d = T_B$

$P = N$

$\int P \cdot dt = \int N \cdot dt$

$\therefore \boxed{\text{Energy} = \int N \cdot dt}$

By adjusting the distance (d) i.e pos^n of permanent magnet w.r.t to centre of the disc speed of the disc is adjusted.

Recovering & Ragistering Mechanism:-

0	0	0	0

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0	0	1	0

0	0	9	9

0	1	0	0

Reduced Gear Train mechanism is used to record the energy consumed by the load over a time period so that energy meter is working as an integrating instrument.

Formulae :-

Energy meter constant $K = \frac{\text{No of revolutions made by disc}}{\text{Kwhr}}$

$$K = \frac{N}{P.t}$$

N = No of Revolutions.
P = power in kW.
t = time in hr.

Energy Recorded or Measured in Kwhr

$$W_m = \frac{\text{Total NO of revolutions}}{K}$$

Total Energy Consumed by load in Kwhr

$$W_T = \frac{VI\cos\phi}{1000} \times \frac{t}{3600}$$

V → voltage in V
I → Load Current in A
Half load → I/2
cos ϕ → Load P.f

t = time duration in secs of conn

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% Error = $\boxed{\% E_r = \frac{W_m - W_T}{W_T} \times 100}$

E_r → +ve → Meter Runs fast.
E_r → −ve → Meter Runs slow.

Pg 42
Q20

$W_m = 90/1800 = 0.05$ Kwhr.

$W_T = \frac{VI\cos\phi}{1000} \times \frac{t}{3600}$, min (convert to sec multiply by 60)

$\Rightarrow \frac{230 \times 10/2 \times 1}{1000} \times \frac{3 \times 60}{3600}$

$\Rightarrow 0.0575$ Kwhr

$\% E_r = \frac{.05 - .0575}{.0575} \times 100$

$\Rightarrow -13.04\% \Rightarrow 13.04\%$ slow

Q4

$K_1 = 14.4$ A sec/rev.

$K = \frac{1}{14.4} \frac{rev}{A sec}$, $V = 250V$

$\Rightarrow \frac{1}{\frac{14.4 \times 250}{1000} \times \frac{1}{3600}} \frac{rev}{Kwhr}$

$\Rightarrow 1000$ rev/Kwhr.

Error & Compensations in Energy Meter –

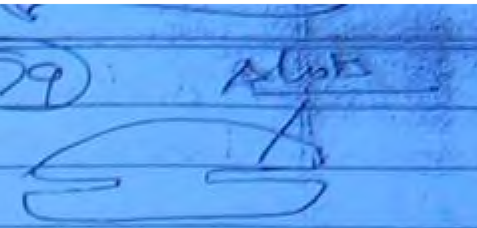
(1) Friction and Low Load compensation →

Shading loop is used to compensate friction and low load condition. whenever the potential coil is excited flux is passing through the shading loop in some part of the loop this area is called shaded area and the remaining part is non magnetized called unshaded area due to this magnetic field difference, the disc start rotating used for compensating friction and low load condition.
The position of the shading loop can be adjusted.

(2) Creeping →

If the friction is over compensated by keeping the shading loop nearer to the potential coil. The disc start rotates with only potential coil is excited and there is no load current.
It is called creeping. It produces a creeping error

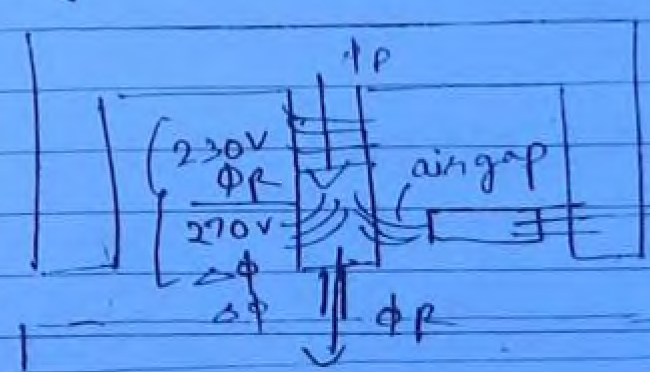
$$\% \text{ Creeping} = \frac{\text{No of revolutions / hr due to creeping}}{\text{No of revolutions / hr due to load}} \times 100$$

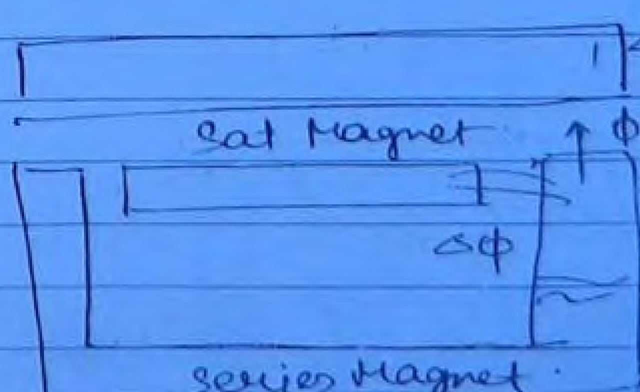


For reducing the creeping, two holes or slot's are kept on opposite side's of rotating disc. If these holes are below the side limbs of the shunt magnet then opposing torque are produced so that the disc rotation stops.

Small iron piece is kept on the disc which is attracted by the permanent magnet so that disc rotation stops.

) Over voltage compensation →





Overload (current)

A saturable shunt magnet is provided between the central limb and side limbs of a shunt magnet which diverts additional flux produced by the additional voltage so that overvoltage is compensated.

Over Current compensation → The saturable magnet is provided between the limbs of a series magnet which divert the additional flux due to overloading hence compensate the overload.

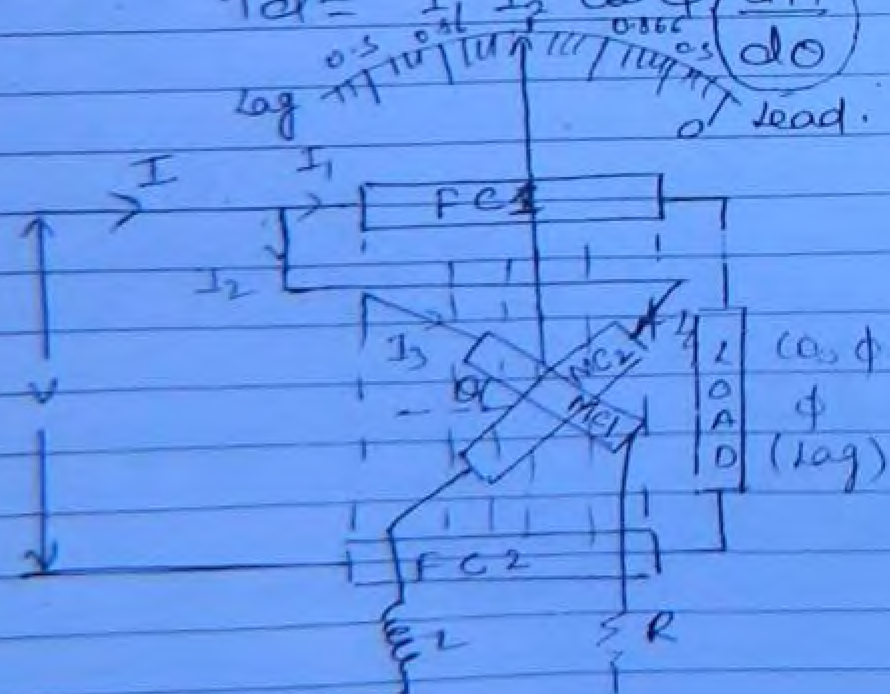
$$\% \text{Creeping Error} = \frac{1 \times 60}{\frac{230 \times 5 \times 1}{1000} \times 2400}$$

$$\Rightarrow \frac{60}{1.15 \times 2400}$$

Power Factor Meter: —

$$T_d = I_1 I_2 \cos\phi \left(\frac{dM}{d\theta}\right)$$

(Scale) UPF

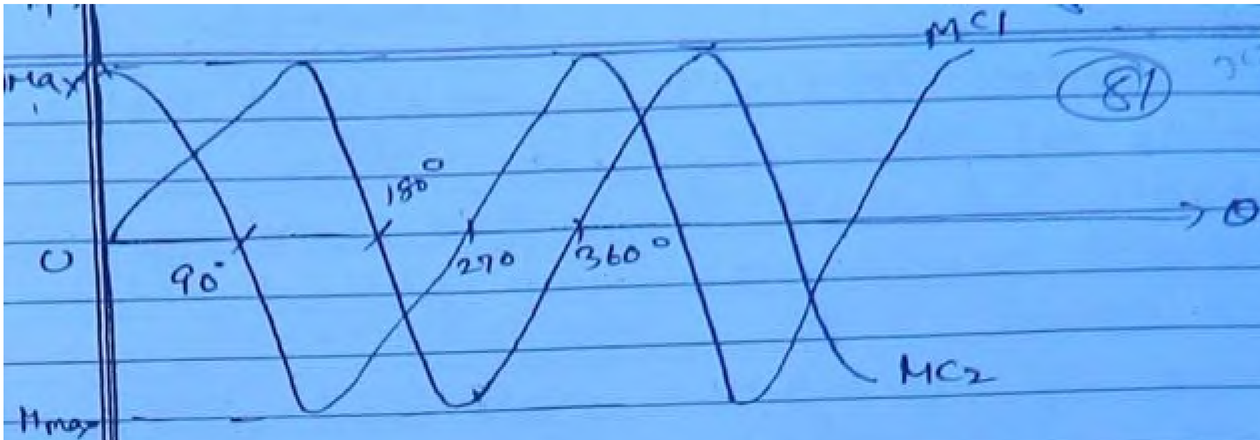


$\theta \propto I^2$

$\theta \propto V$

$\theta \propto P$

$\theta \propto \phi$ $(\theta = \phi)$



MC1

$M = M_{max} \cos\theta$

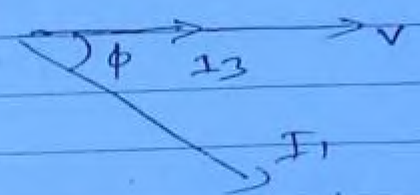
$\left|\frac{dM}{d\theta}\right| = M_{max} \sin\theta$

MC2

$M = M_{max} \sin\theta$

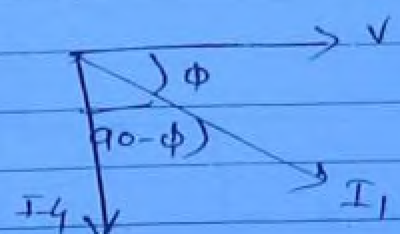
$\frac{dM}{d\theta} = M_{max} \cos\theta$

MC1 →



$T_d = I_1 I_3 \cos\phi \; M_{max} \sin\theta$

MC2 →



$T_{d_2} = I_1 I_4 \cos(90-\phi) \cdot M_{max} \cos\theta$

$T_{d_2} = I_1 I_4 \; M_{max} \sin\phi \cdot \cos\theta$

At balance,

$\boxed{T_{d_1} = T_{d_2}}$; Assume $|I_3| = |I_4|$

$\cancel{I_1} \cancel{I_3} \cancel{M_{max}} \sin\theta \cos\phi = \cancel{I_1} \cancel{I_4} \cancel{M_{max}} \sin\phi \cos\theta$

$\tan\theta = \tan\phi$

$\Rightarrow \boxed{\theta = \phi}$

Power factor meter is working on the principle of electrodynameter which consist of two fixed coil and two moving coil's and the moving coil are placed ⊥ to each other

(82)

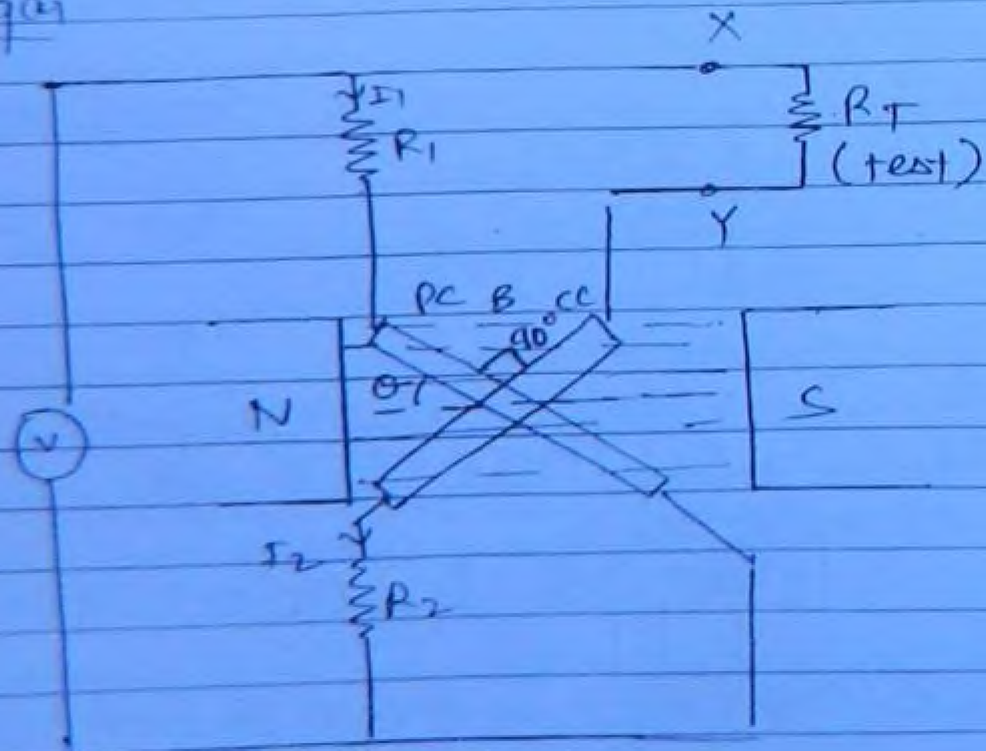
In case of 3φ system polarised vane power factor meter is used it also works on same principle.

There is no external control torque provided by this spring because the two moving coil's are balanced each other inside the magnetic flux.

Damping torque is provided by the air friction.

Application →

1) Meggar



- Megger is a most practical instrument used for measurement of high resistance of insulation of cables and winding insulation of electrical motor, generator and transformer.
- control torque is provided by spring to bring pointer to zero initial position.
- damping torque is provided by air friction
- Hand driven DC generator is used for providing the supply to the current coil and potential coil.

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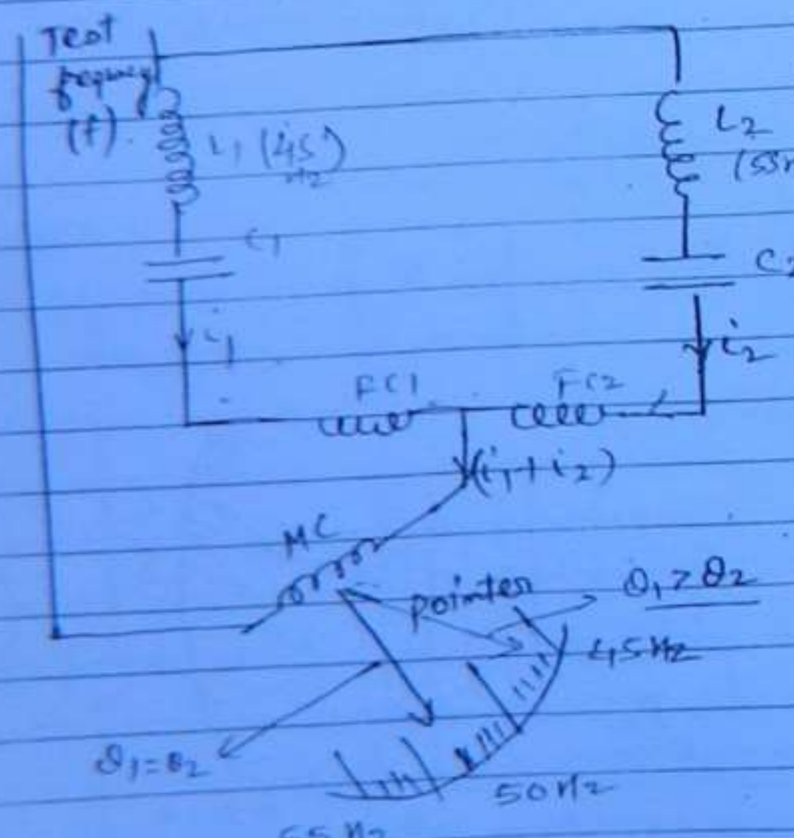
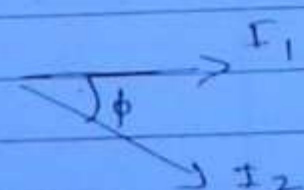
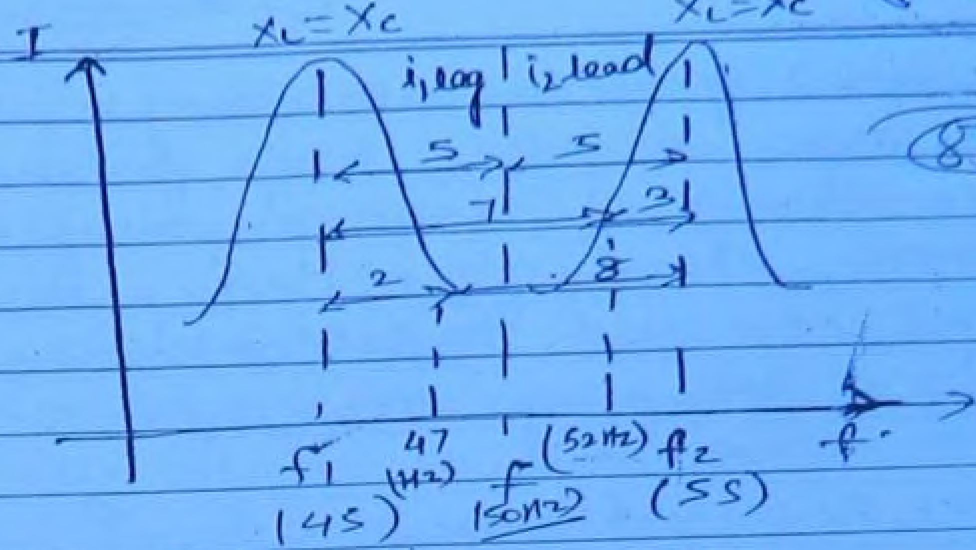
Power Frequency Meter →



→ Resonant freq CRt.

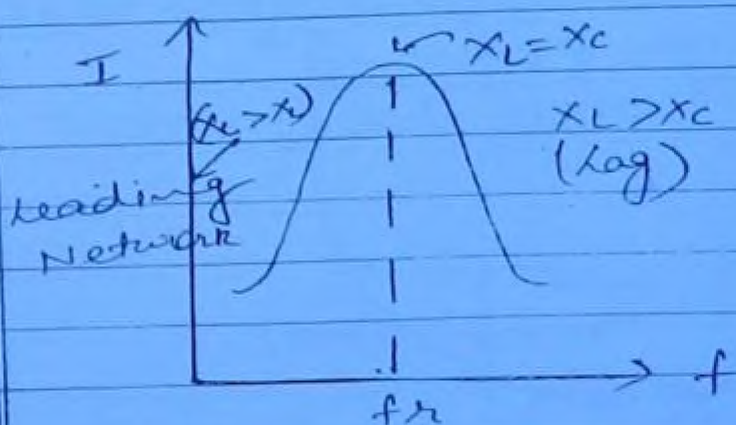


$$T_d = \underset{F.C}{I_1}\, \underset{M.C}{I_2} \cos\phi \cdot \left(\frac{dM}{d\theta}\right)$$



$$f_1 = \frac{1}{2\pi\sqrt{L_1C_1}} = 45\,Hz,$$

$$f_2 = \frac{1}{2\pi\sqrt{L_2C_2}} = 55\,Hz,$$



$\underline{f < f_r}$

$\uparrow X_C = \frac{1}{2\pi f\downarrow C}$

$\downarrow X_L = 2\pi f\downarrow L$

$\underline{f > f_r},\quad \downarrow X_C = \frac{1}{2\pi f\uparrow C},\quad \uparrow X_L = 2\pi f\uparrow L$

(1) $f = 50\,Hz$

$$f - f_1 = f - f_2$$
$$5 = 5$$

(2) $f = 52\,Hz \quad \theta_1 = \theta_2$

$$(f - f_1) > (f_2 - f)$$
$$\theta_1 > \theta_2$$

(3) $f = 47\,Hz \quad (f - f_1) < (f_2 - f_1)$

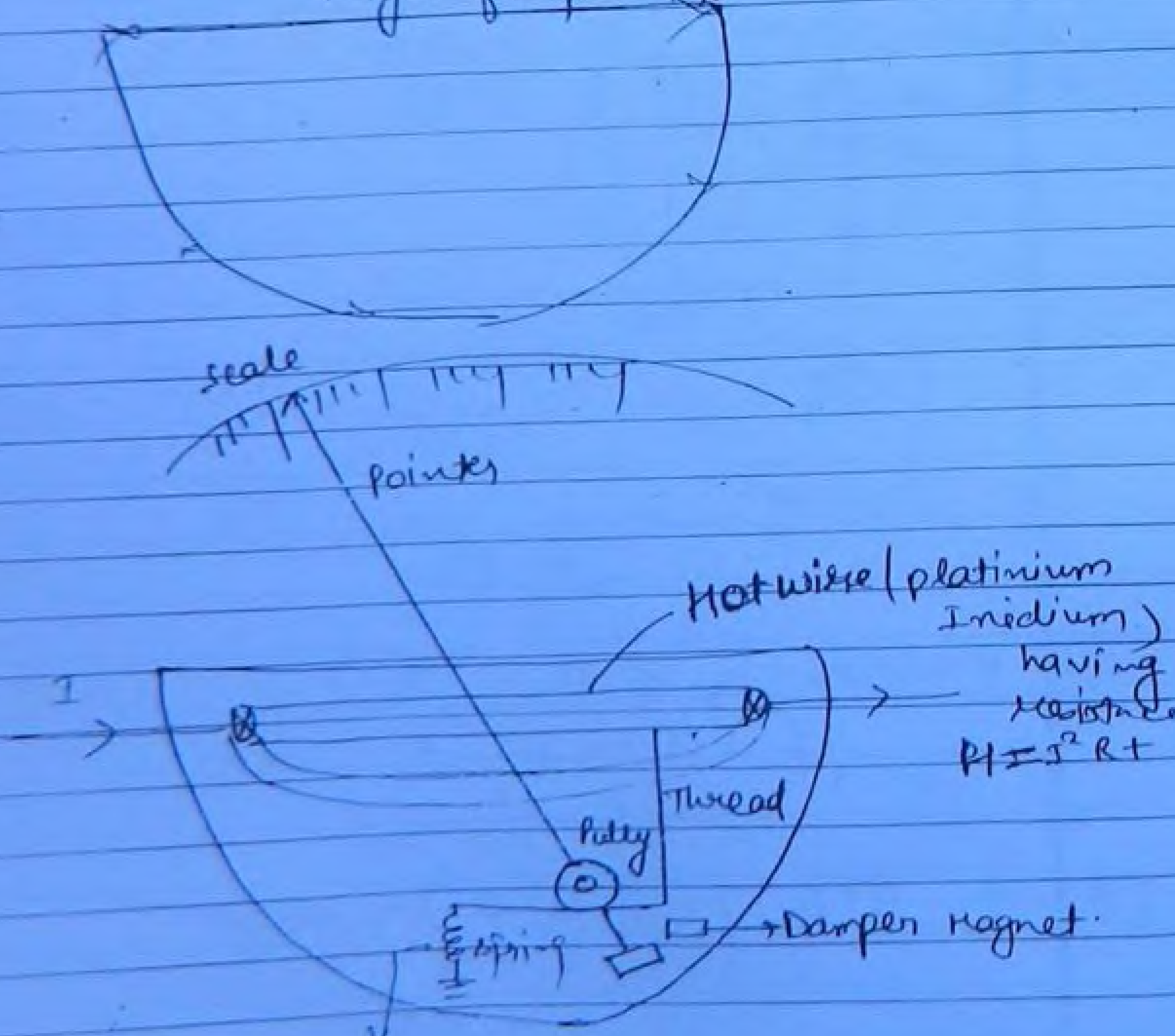
$$2 < 8$$
$$\theta_1 < \theta_2$$

Power frequency meter working on a principle of electrodynammeter is used to measure the frequency in the power frequency range. Small weight is kept on spindle which provide the controlling torque. Air friction damping is used.

Thermal Meters :- [Heating Effect] (86)

(i) Hot wire meter
(ii) Thermo Couple meter.

(i) Hot wire Meter → [Measurement of current at high frequency]



Used for controlling tor...

Hot wire Meter working on a principle of heating effect. Whenever, the dc or high frequency current is passing through hot wire due to I^2R losses internal heat is produced so that hot wire expand's. The expansion of the hot wire is driving the pully which in turn moves the pointer which indicate the current reading.

82

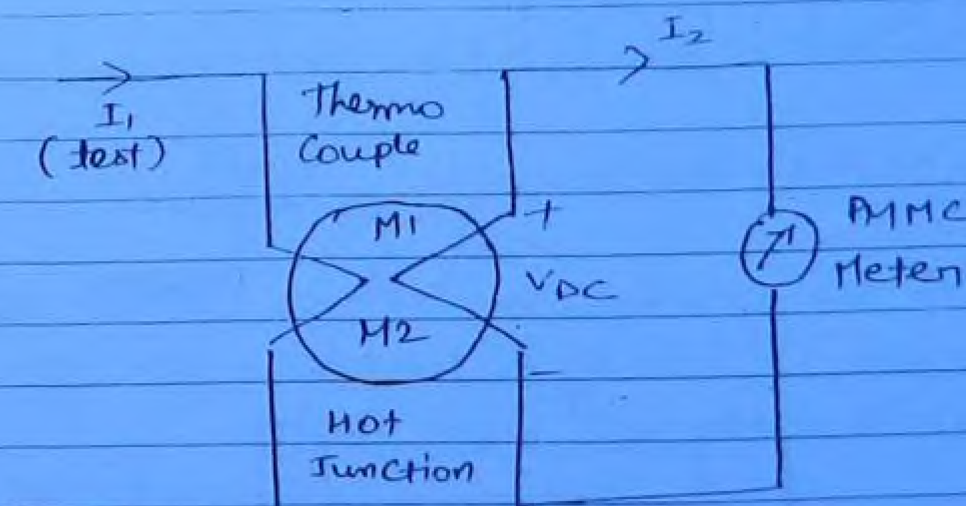
Application →

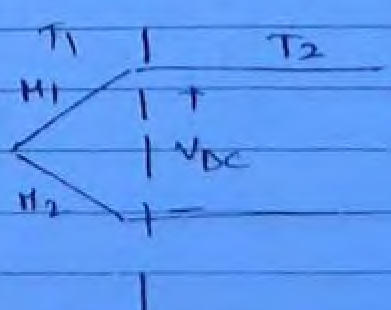
Used for measurement of high frequency current

~~over~~ Disadvantage →

power loss is more, Slow in operation, not suitable for overloading.

(ii) Thermo Couple Meter →





Seeback effect

The thermo couple meter working on a principle of seeback effect.

Seeback effect → If two metal forming a junction and they are operating at two different temp^s the dc potential is produced across the junction. This is measured by using PMMC meter in case of thermocouple meter.

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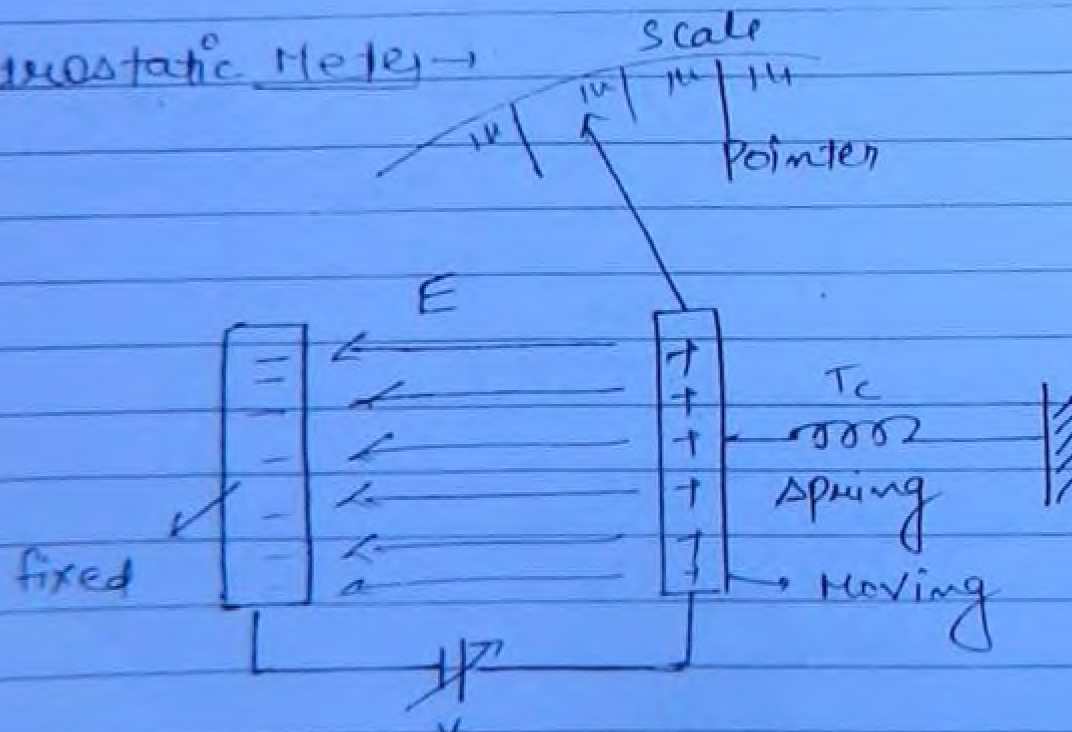
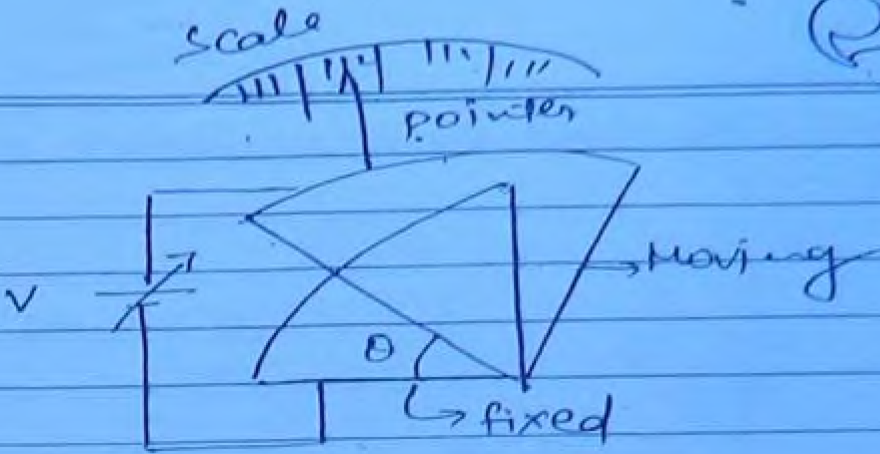
Application →

Used to measure both dc & ac in case of ac measures RMS Qty of current. The

- PMMC instrument is calculated to measure RMS Quantity.
- It is used to measure upto 50MHz of i/p supply frequency.
- It is not suitable for overloading condition.

Electrostatic Meter →





MI Meter →

$$\boxed{T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}}$$

Electrostatic →

$$T_d = \frac{1}{2} V^2 \frac{dC}{d\theta} = K\theta$$

$\boxed{\theta \propto V^2}$ < AC (RMS), DC

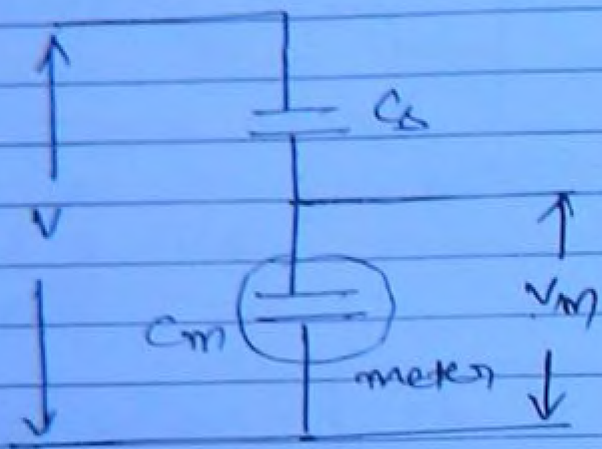
Electrostatic meter working on the principle of change of electrical field (electrostatic energy) i.e $\frac{1}{2}CV^2$ b/w the two plates of the capacitor.

→ Spring is used for providing controlling torque

→ Fluid friction damping is used which provides damping torque.

Application →
used for measurement of higher voltages upto 220 kV.
It is provided with series [illegible]

capacitance for the enhancement of electrostatic voltmeter.



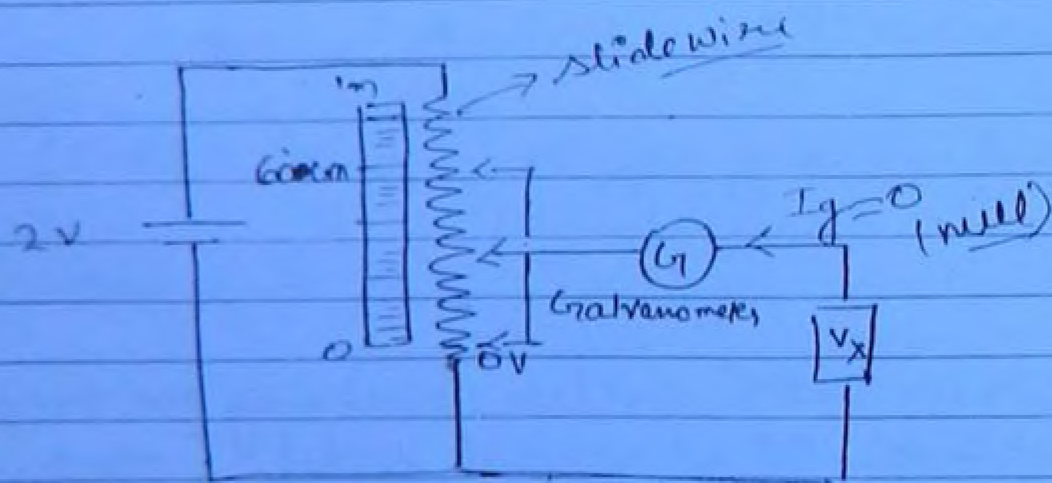
$$\frac{V}{V_m} = m$$

$$C_s = \frac{C_m}{(m-1)}$$

C_s = series multiplier capacitance.

C_m = Meter Capacitance

Potentiometer → or Null Detector.



1m = 2 V

100 cm = 2 V

$\boxed{1\text{cm} = .02\text{ V}}$

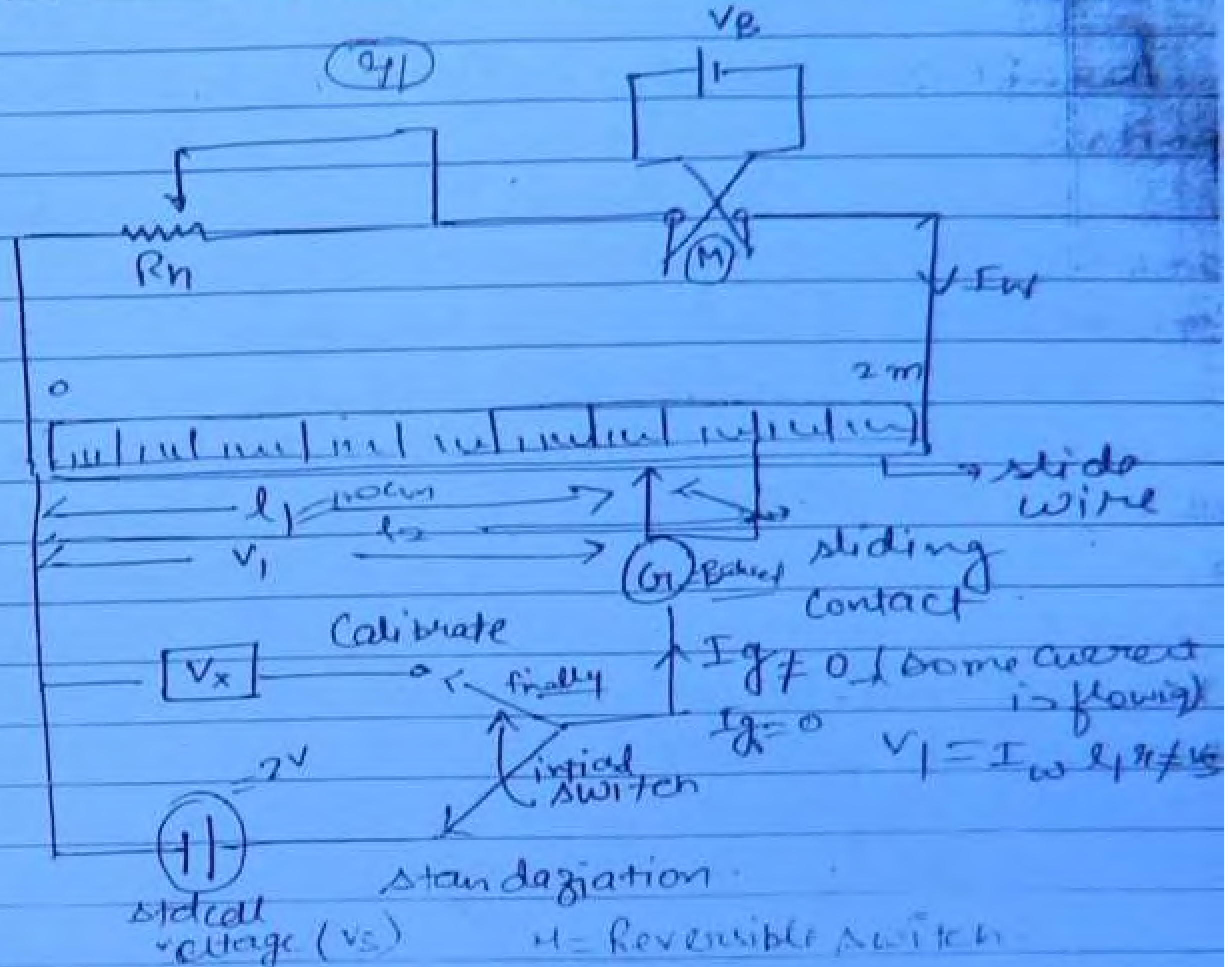
V_x = Test cell

$V_1 = 60 \times .02$

→ 1.2 Volt.

$\boxed{V_x = 1.2\text{ Volt}}$

Practical Potentiometer :-



l = Length of slide wire.

r = Resistance of slide wire in Ω/m.

l_1 = total resistance of slide wire.

Iw = Working current

$$\boxed{I_w = \frac{V_B}{R_h + l_1 r}} \quad \text{— (1)}$$

V_B → constant

$l_1 r$ → const

R_h → variable

$\cancel{V_1} = \cancel{I_w} l_1 r = V_s$

$V_1 \neq V_s$ so, also change V

$I_g \neq 0$

Bridge Balanced will fail

Switch at standization →

potentiometer is balanced ($I_g = 0$)

$V_s = I_w l_1 r$

$$\boxed{I_w = \frac{V_s}{l_1 r}} \quad \text{— (2)}$$

Switch at Calibrate:-

potentiometer is balanced ($I_g = 0$) at l_2

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$$V_x = I_w \cdot l_2 r$$

$$\boxed{I_w = \frac{V_x}{l_2 r}} \quad -(3)$$

(2) = (3)

$$\frac{V_s}{l_1 r} = \frac{V_x}{l_2 r}$$

$$\boxed{V_x = V_s \frac{l_2}{l_1}}$$

Standardization → The process of converting Cm scale in terms of voltage is called standardization.

For this process std cell or known voltage source is used.

once the instrument is standardizse the working (I_w) never be changed.

A reversible switch (M) is used to nullify the effect of thermo electric emf.

Power consumption is very low bcoz the reading is taken at the null position of the galvanometer.

It is more accurate instrument compare to the deflection type of meter's for static measurement

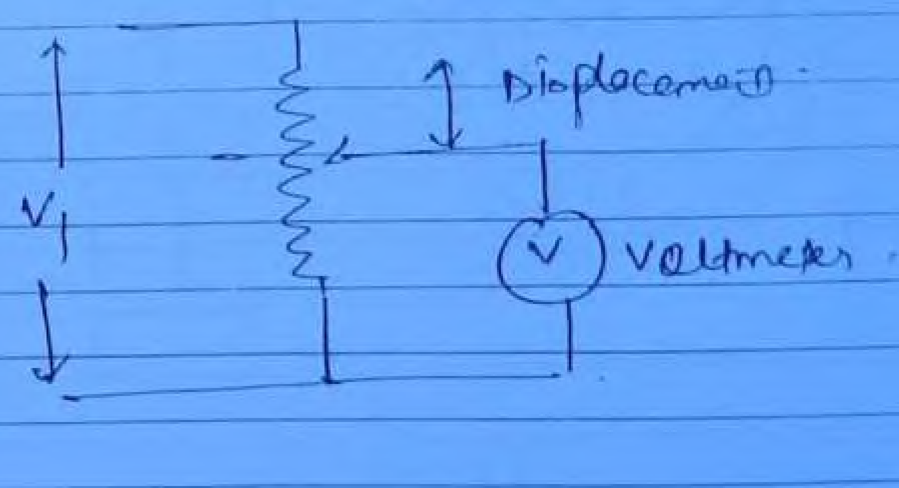
Sensitivity is very high compare to deflecting meter

→ not suitable for measurement of dynamic data (93)

→ For these type deflecting meter are used.

→ Response is very slow compare to deflecting meters

→ In case of ac measurement electrodynamic meter. Ammeter is used as a null detector it is used for measurement of non electrical quantity like displacement, pressure, force etc along with a voltmeter is connected for indicating the electrical output. The voltmeter should have high resistance to reduce the loading effect otherwise error & accuracy are affected. It is also used as a Comparator (summing device) in case of control system application



n=3
21

$V_s = 1.18 V$

$l_1 = 600 mm$

$l_2 = 680 mm$

(94)

$V_x = ?$

$$V_x = \frac{V_s l_2}{l_1}$$

$$\Rightarrow 1.18 \times \frac{680}{600}$$

$$\Rightarrow 1.34 \text{ volt.}$$

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$70 cm = 1 \Omega$

$1 cm = 1/70 \Omega$

$50 cm = 50/70$

$V = I_w \cdot R$

$1.45 = I_w (50/70)$

$I_w = 2.03 A.$

Instrumentation Transformer →

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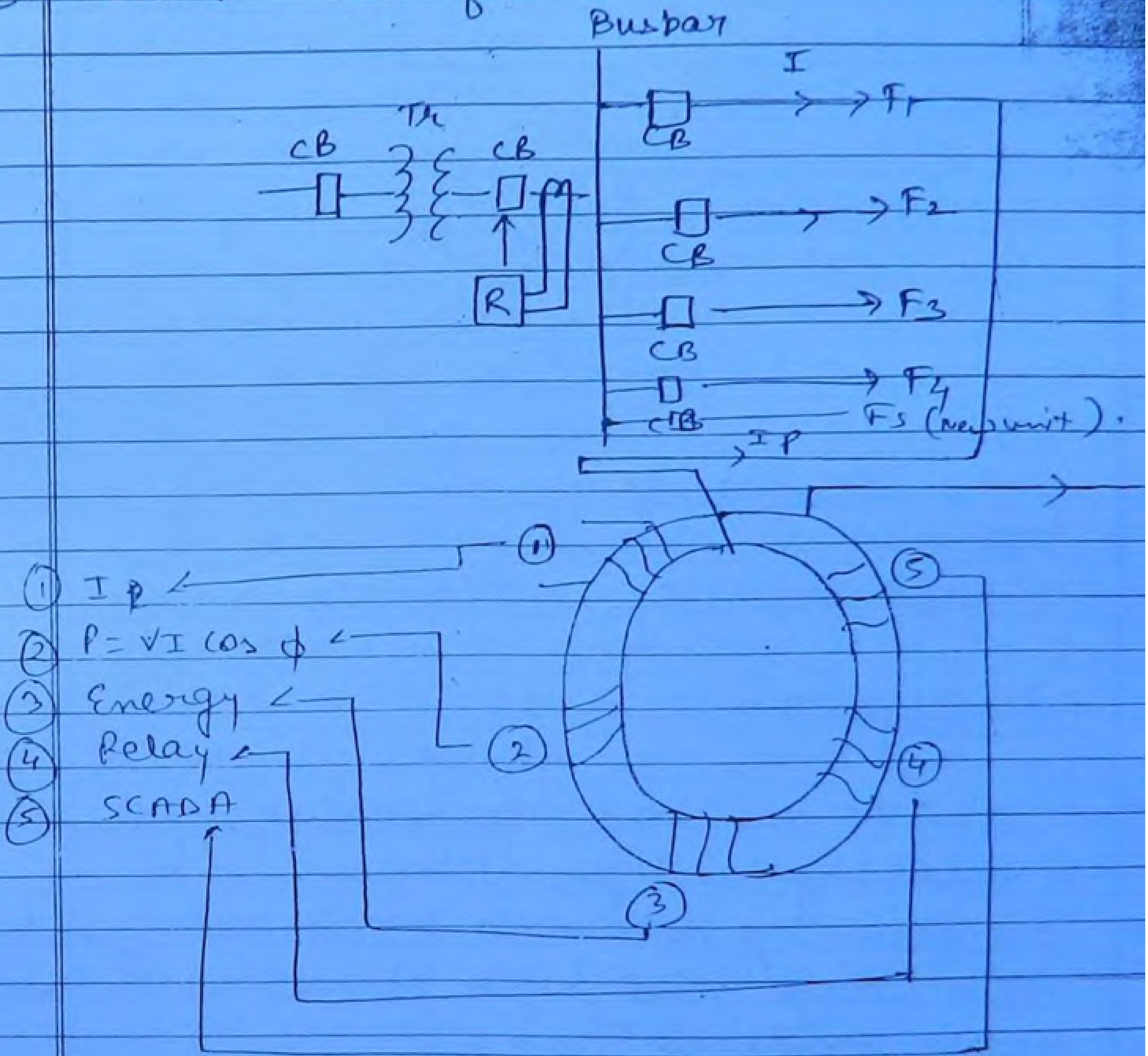
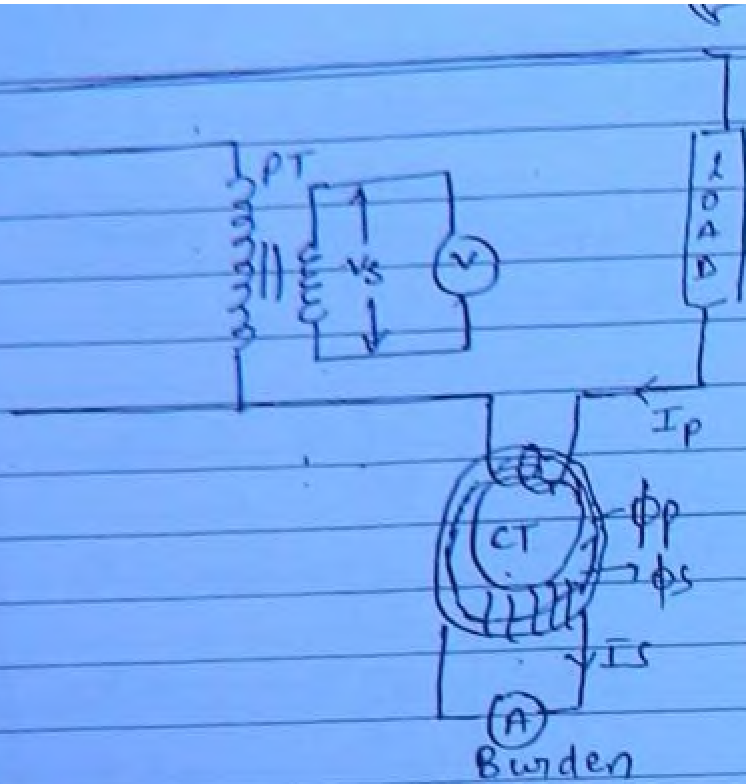
1. Current Transformer (CT)
2. Potential Transformer (PT)



Available : (200/400/600/800) / 1A or 5A
(Ip above 200/400/600/800; Is above 1A or 5A)

Adopted : 400/1A
↓ 600/1A.



96

P.T

Rated Secondary $V_s = 110V$.

CT

Rated Secondary $I_s = 1A$ or $5A$.

Advantages →

(i) CT and PT are used for multiple application's ie measuring and protection.

(ii) The measuring instrument is isolated from the higher voltage and higher current so that need not be insulated.

(iii) ~~Over~~ power consumption is very low

(iv) Higher rating of voltage and current's are reduced to lower voltage and current's.

(v) Accuracy is higher.

(vi) Maintenance, replacement is easier.

Current Transformer →

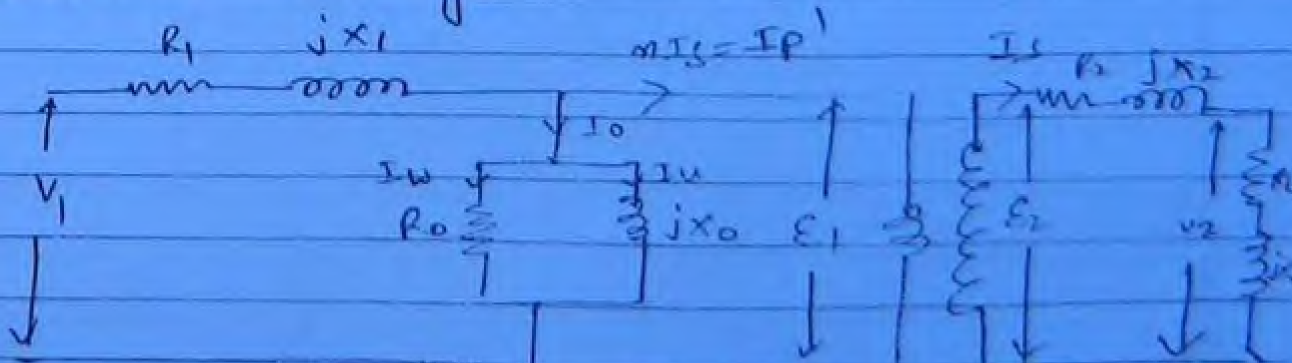
(92)

Effect of secondry open →

The current Xmer is connected in series to the load. Secondry side is connected to the measuring instrument like ammeter, relay etc. If primary is excited the primary flux ϕ_p will set up in the core which induces a voltage in the secondry winding if the burden is connected on secondary than secondry current I_s flows produces a flux ϕ_s which opposes the ϕ_p.

→ If the secondary burden is removed i.e open circuited than ($I_s = 0$) and ($\phi_s = 0$) so that there is no opposing flux to ϕ_p and I_p will not be changed because it is entirely depending on load of the system. The high value of ϕ_p induces high voltage in the secondry which causes insulation damage of the secondary winding and hence secondary of the CT never be open circuited if the primary is excited.

Equivalent ckt of CT →



Turns Ratio $= n = \dfrac{N_2}{N_1}$

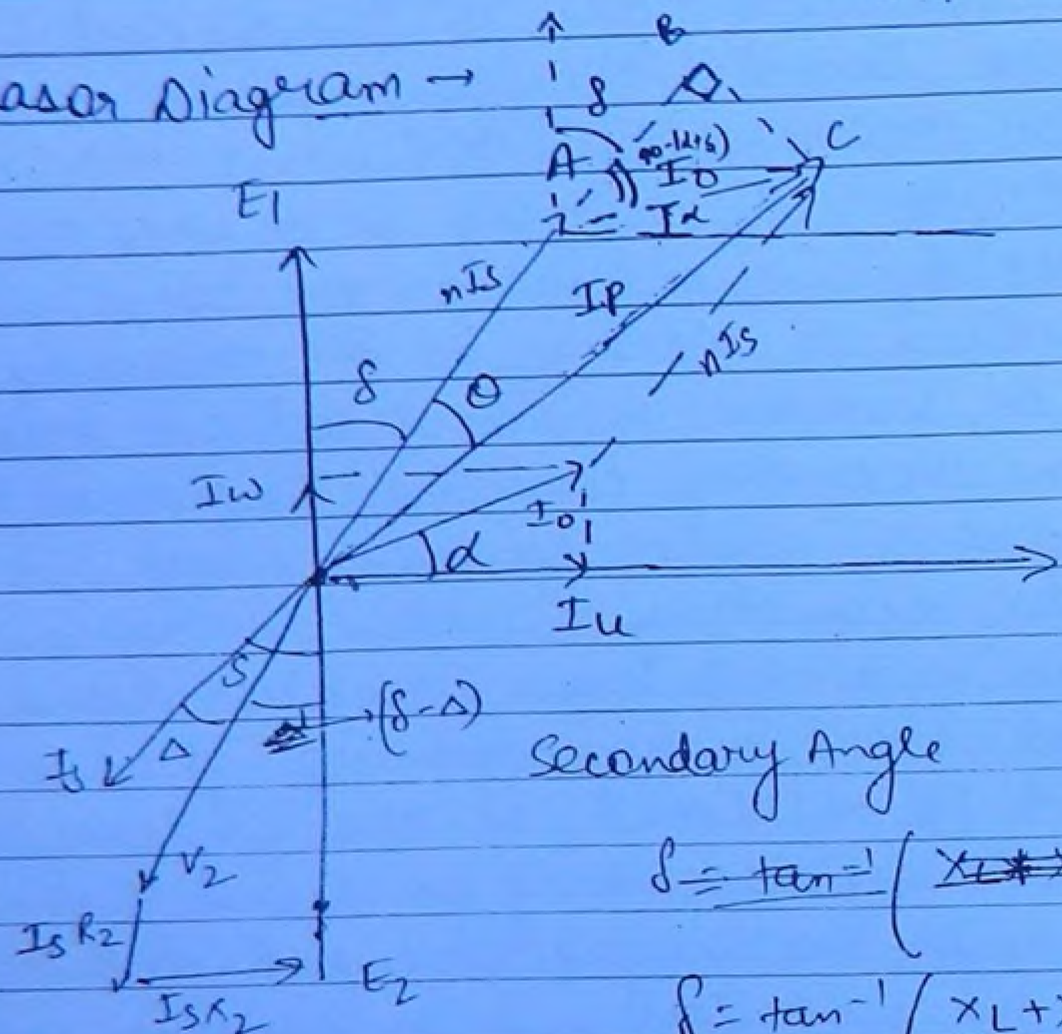
Burden Impedance.

$Z_L = R_L + jX_L$.

98

$\Delta = \tan^{-1}\left(X_L / R_L\right)$

Phasor Diagram →



Secondary Angle

$\delta = \cancel{\tan^{-1}\left(X_L * X_2\right.}$

$$\delta = \tan^{-1}\left(\frac{X_L + X_2}{R_L + R_2}\right)$$

Nomianal Ratio -

$$K_n = \frac{I_p}{I_s} - \frac{N_2}{N_1} = n.$$

Error's in Instrument xmer's →
no load component of current I_0 [illegible]
and phase angle error are produced [illegible]
case of CT's & PT's. (99)

(1) Ratio Error → The transformer turns ratio must be equal to the current ratio in case of ideal condition.

$$n = \frac{N_2}{N_1} = \frac{I_P}{I_S} \quad [\text{Ideal}].$$

But in practical, CT's $I_P/I_S = n + I_0/I_S$
Due to this the turns ratio is not equal to current ratio which produces an error called ratio Error.

$$r = \frac{\text{Nominal Ratio}(K_n) - \text{Actual ratio}(R)}{\text{Actual Ratio }(R)} \times 100$$

$$\boxed{\%r \Rightarrow \frac{K_n - R}{R} \times 100}$$

$K_n \simeq n = N_2/N_1$
$R = I_P/I_S$

$AB = I_0 \cos[90 - (\alpha+\delta)]$
$AB \Rightarrow I_0 \sin(\alpha+\delta)$

$BC = I_0 \sin[90 - (\alpha+\delta)]$
$\boxed{BC = I_0 \cos(\alpha+\delta)}$

$I_p = OC$
$OC^2 = OB^2 + BC^2$
$OC^2 = (OA + AB)^2 + BC^2$

$$I_p^2 = [nI_s + I_o \sin(\alpha+\delta)]^2 + I_o^2 \cos^2(\alpha+\delta)$$

$$\boxed{I_p \approx nI_s + I_o \sin(\alpha+\delta)}$$

$$R = \frac{I_p}{I_s} = n + \frac{I_o \sin(\alpha+\delta)}{I_s}$$

$$= n + \frac{I_o \sin\alpha \cos\delta + I_o \cos\alpha \sin\delta}{I_s}$$

$$\boxed{R = n + \frac{I_w \cos\delta + I_u \sin\delta}{I_s}}$$

(i) Secondary is pure resistive burden $\delta = 0^\circ$

(1) I_w is given

$$R = n + \frac{I_w \cos 0}{I_s} \Rightarrow n + \frac{I_w}{I_s}$$

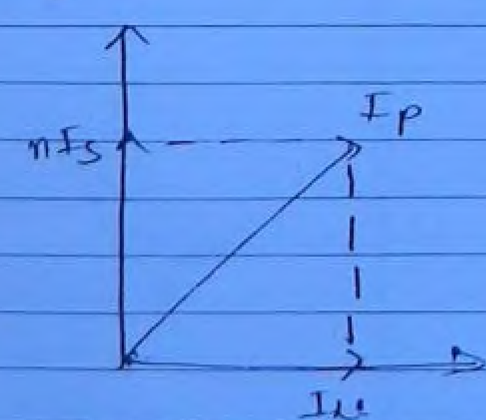
$$\boxed{\%\sigma = \frac{n-R}{R} \times 100}$$

(ii) I_u is given :- Calculate I_p from phasor.

$$\cancel{R = n + I_u \sin}$$

$$I_p^2 = I_u^2 + n^2 I_s^2$$

$$\boxed{I_p = \sqrt{I_u^2 + n^2 I_s^2}}$$



$$R = \frac{I_p}{I_s} = \frac{\sqrt{I_M^2 + n^2 I_s^2}}{I_s}$$

(d)

$$\boxed{\% \sigma = \frac{n - R}{R} \times 100}$$

Ch-9

Q1

$N_1 = 1$, $500/5 \Rightarrow 100 = n$.

$I_s = 5$ amp

$\delta = 0°$

$I_u \cdot N_1 = 250$ AT

$I_u = \frac{250}{1} \Rightarrow 250A$.

$$R = \frac{\sqrt{I_u^2 + n^2 I_s^2}}{I_s}$$

$$\Rightarrow \frac{\sqrt{(250)^2 + (100)^2 (5)^2}}{5} \Rightarrow 111.8$$

$$\Rightarrow \frac{100 - 111.8}{111.8} \times 100$$

$$\sigma \Rightarrow -10.56\%$$

10)

Phase Angle Error → In ideal xmer, the phase angle difference b/w I_p & I_s is 180° but in practical xmer this angle is less by θ due to no load component I_0.

$$\tan\theta = \frac{BC}{OB} = \frac{I_0 \cos(\alpha + \delta)}{n I_s + I_0 \sin(\alpha + \delta)}$$

Assume $n I_s > I_0 \sin(\alpha + \beta)$, $\tan\theta \approx \theta$

$$\theta = \frac{I_0 \cos(\alpha+\delta)}{n I_s}$$

$$\Rightarrow \frac{I_0 \cos\alpha \cos\delta - I_0 \sin\alpha \sin\delta}{n I_s}$$

$$\boxed{\theta = \frac{I_\mu \cos\delta - I_w \sin\delta}{n I_s}} \text{ radians}$$

$$\boxed{\theta = \frac{I_\mu \cos\delta - I_w \sin\delta}{n I_s} \times \frac{180}{\pi}} \text{ degrees.}$$

Pure Resistive Burden. $\delta = 0$.

$$\boxed{\theta = \frac{I_\mu}{n I_s} \times \frac{180}{\pi}} \text{ degree.}$$

$\Rightarrow$ phase angle difference between. I_p & I_s is $\underline{180 - \theta}$

Qn-9
Q2

$N_1 = 1$ $\quad n = \frac{N_2}{N_1} \rightarrow 500.$

$N_2 = 500$

$I_s = 5A.$

$\delta = 0°$

$I_\mu N_1 = 200$

$I_\mu = \frac{200}{1} \Rightarrow 200,$

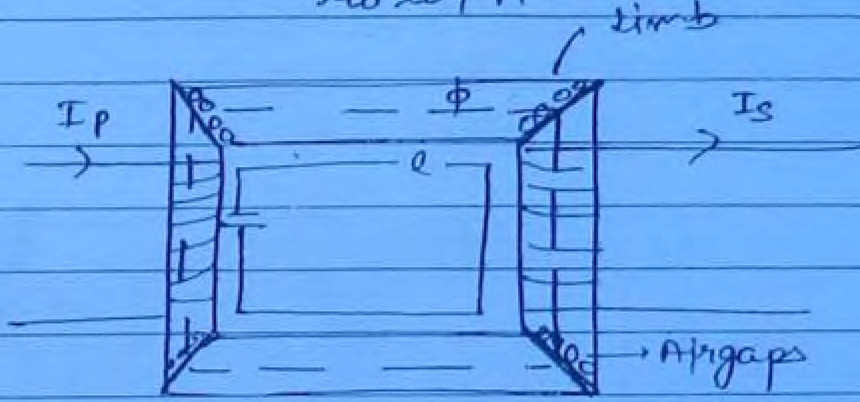
$$\theta = \frac{I_\mu}{n I_s} \times \frac{180}{\pi} = \frac{200}{500 \times 5} \times \frac{180}{\pi} = 4.6°$$

I_p & I_s angle $= 180 - 4.6 \Rightarrow \underline{175.4°}$

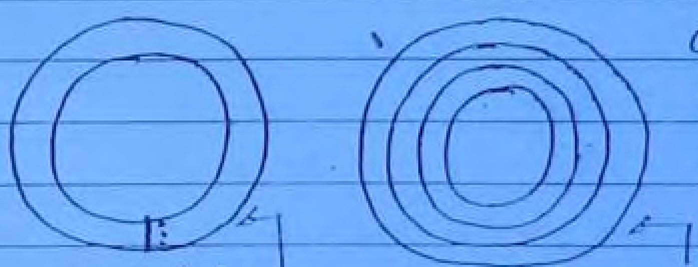
Methods of Reducing Errors

(1) Reducing the Reluctance →

$$S = \frac{l}{\mu_0 \mu_r A}$$



we have to reduce these air gaps. so we use Toroidal core and Strip wound core.



Toroidal core Strip wound core

Core Materials

Hypernik
Per mendur
Silicon steel
} Higher.

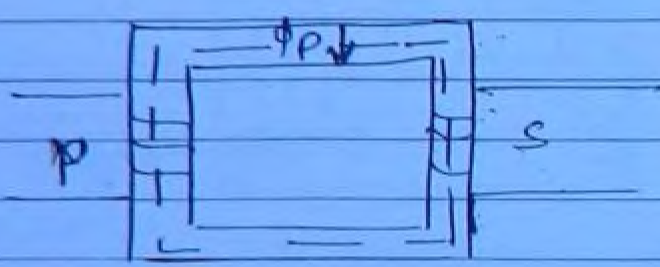
→ If the no load component I_0, is maintain low value than δ & θ are maintain low value.

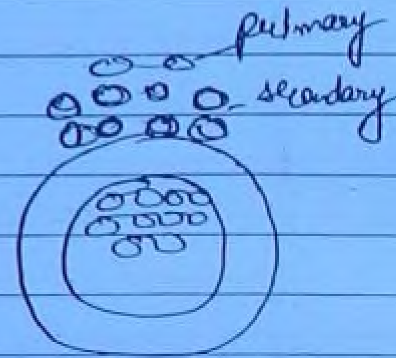
To reduce I_0, the reluctance of magnetic path is reduced by reducing the no of core joints and hence the ~~distance~~ core is of ~~between~~ toroidal or strip wound core of circular shape is used.

(ii) Reducing the distance b/w primary & secondary winding →

(IO4)

The air leakage flux is reduced with the distance b/w primary & secondary is maintained as low as possible so that Io is maintained low value.





3) $n = \frac{\cancel{N_2}}{\cancel{N_1}} = \frac{\cancel{I_P}}{\cancel{I_S}}$

$\cancel{n I_S = I_P = N_2/N_1}$

(iii) Reducing Primary Turn's →

$$n = \frac{N_2}{N_1} = \frac{I_P}{I_S}$$

$$n I_S = \uparrow I_P' = N_2/N_1 \downarrow \quad N_1 = 1$$

$$\downarrow \theta = \frac{I_0 \cos(\alpha + \delta)}{n I_S} = \frac{I_0 \cos(\alpha + \delta)}{I_P' \uparrow}$$

$$R = n + \frac{I_0 \sin(\alpha + \delta)}{I_S} \downarrow$$

$$R = n + \frac{I_0 \sin(\alpha + \delta)}{I_P' \uparrow} \downarrow$$

By maintaning single turn primary, I_p' is maintained higher so that σ and θ are reduced.

(105)

(IV) Turns Compensation → It is used only for reducing ratio error by reducing the secondary turns compare to the physical no of turns available on the secondary.

$$\sigma = \frac{n-R}{R}$$

$$R = n + \frac{I_0 \sin(\alpha+\delta)}{I_s}$$

$$R = n + K_1$$

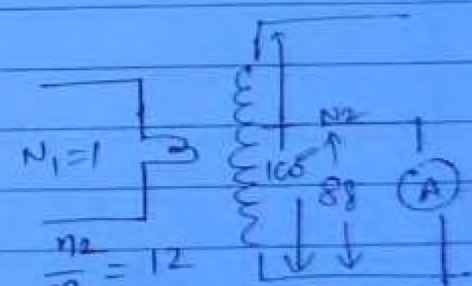
$$R = n - K_1 + K_1$$

$$R = n$$

$$\sigma = 0$$

$$n = \frac{500}{5} \Rightarrow 100$$

$$N_1 = 1, \quad N_2 = 100$$



$$R = 111.8 = 112$$

$$R = 112$$

$$= 100 + 12$$

$$= 100 - 12 + 12$$

$$= 88 + 12 = 100$$

$\frac{n_2}{n_1} = 12$

$n_1 = 1$

$n_2 = 12$

$\sigma = 0$

$$R = n$$

$$\boxed{\sigma = 0}$$

With this method θ is not affected only σ is reduced.

v) WILSON COMPENSATION METHOD → An addititional auxilary short circuit turn is added in series to the secondary winding which causes additional phase lag in the secondary so that the phase angle between the primary and secondary is maintain nearer to 180° and hence the phase angle error is reduced.

106

Measurement of RLC :-

AC Bridges :-

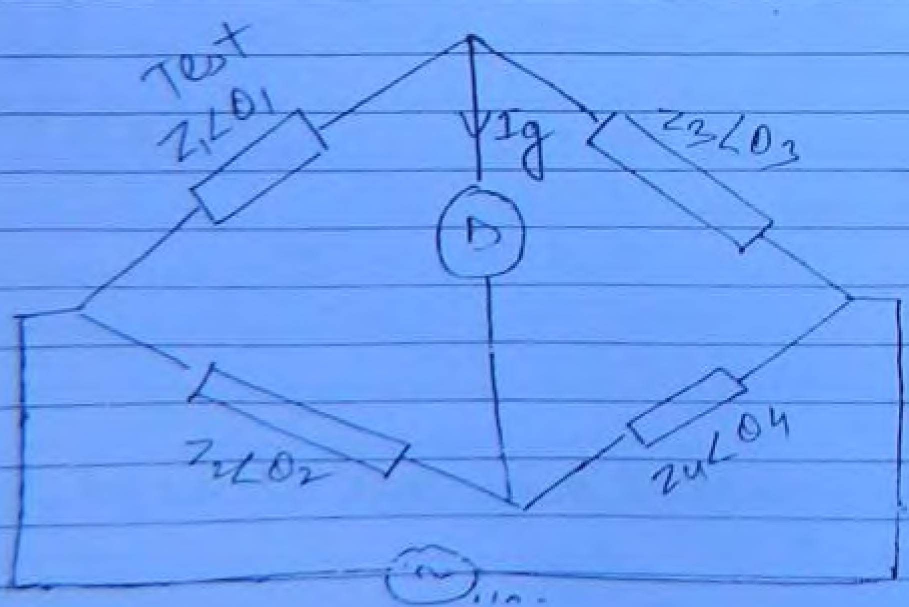
→ used for Measuring :-

- L, C
- ε_r, μ_r
- f

→ Dissipiation factor (D)

→ Loss Angle ($\tan\delta$)

→ Quality factor (Q)



AC Bridges - Detector (D)

(107)

1. Vibration Galvanometer — 5 Hz – 200 Hz
2. Telephone Detector — 200 Hz – 2 KHz
3. Tuned Amplifier — > 2 KHz.

DC Bridges - Detector

(i) Galvanometer —

Bridge is Balanced:- (Conditions)

(i) $|Z_1||Z_4| = |Z_2||Z_3|$ and.

(ii) $\theta_1 + \theta_4 = \theta_2 + \theta_3$

Test :-

$$|Z_1| = \frac{|Z_2|\,|Z_3|}{|Z_4|}$$

$$\theta_1 = (\theta_2 + \theta_3) - \theta_4$$

$$Z_1 = |Z_1| \angle \theta_1$$

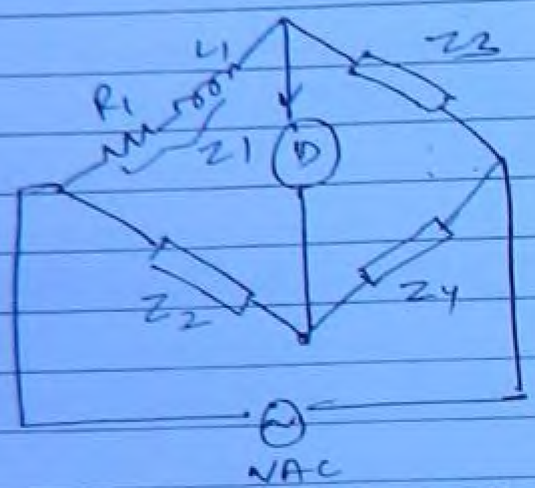
$$\downarrow$$

$$\boxed{Z_1 = \left|\frac{Z_2 Z_3}{Z_4}\right| \angle \theta_2 + \theta_3 - \theta_4}$$

θ_1	Components of Z_1
$0°$	R
$90°$	L
$-90°$	C
$0 < \theta_1 < 90$	R, L
$-90° < \theta_1 < 0$	R, C

Q Consider following statement regarding balance ac bridge shown in the given figure for the measurement of the coil Z_1

(108)



(1) $Z_2 = R_2$ in series with L_2, $Z_3 = R_3$, $Z_4 = R_4$
(2) $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4 \| L_4$
(3) $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$ in series with L_4.
(4) $Z_2 = R_2 \| L_2$, $Z_3 = R_3$, $Z_4 = R_4$.

a) 1 and 2 b) 1 and 4 c) 2 and 3 d) 3 and 4

A: $Z_1 = R_1 + j\omega L_1 \Rightarrow Z_1 = |Z_1| \angle \theta_1$; $\theta_1 = +ve$

$\theta_1 = \theta_2 + \theta_3 - \theta_4$ $Z_1 = \sqrt{R^2 + \omega^2 L_1^2} \angle \tan^{-1}\frac{\omega L_1}{R_1}$

(1) $\theta_1 = +ve$ $|Z_1| \angle \theta_1$, $\boxed{\theta_1 = +ve}$

$\theta_2 = R_2 + j\omega L_2 \Rightarrow +ve$, $\theta_3 = R_3 \Rightarrow \tan^{-1}\left(\frac{0}{R_3}\right) = 0$,

$\theta_4 = 0$.

$\theta_1 = \theta_2 + \theta_3 - \theta_4 = +ve + 0 - 0 = +ve$.

(2) $\theta_2 = 0$, $\theta_3 = 0$, $Z_4 = \frac{R_4 \times j\omega L_4}{R_4 + j\omega L_4} = 90 - \tan^{-1}(\ ) = +ve$

$\theta_1 = 0 + 0 - (+ve)(\theta_4 = +ve)$.

$\rightarrow -ve$

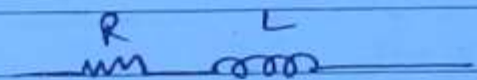
(3) $\theta_2 = \theta_3 = 0$, $\theta_4 = +ve$, $\theta_1 = -ve$

Measurement of Inductance (L) :-

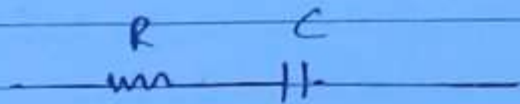
(109)

(i) Maxwell L-C bridge - Medium Q coils ($K < Q < 10$)
(ii) Hays bridge - High Q coils ($Q > 10$)
(iii) L Anderson bridge - Low Q Coils ($Q < 1$)
(iv) Owens bridge - Incremental Inductance (Ua)
(5) Micro

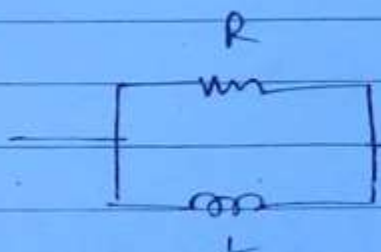
Q factor or Quality factor (Q) :-



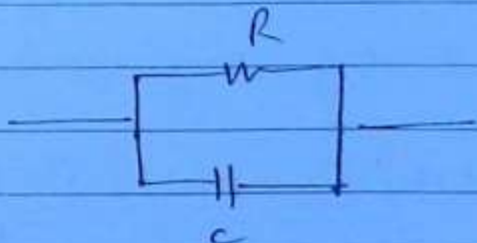
$$Q = \frac{X_L}{R} = \frac{\omega L}{R}$$



$$Q = \frac{X_C}{R} = \frac{1}{\omega C R}$$



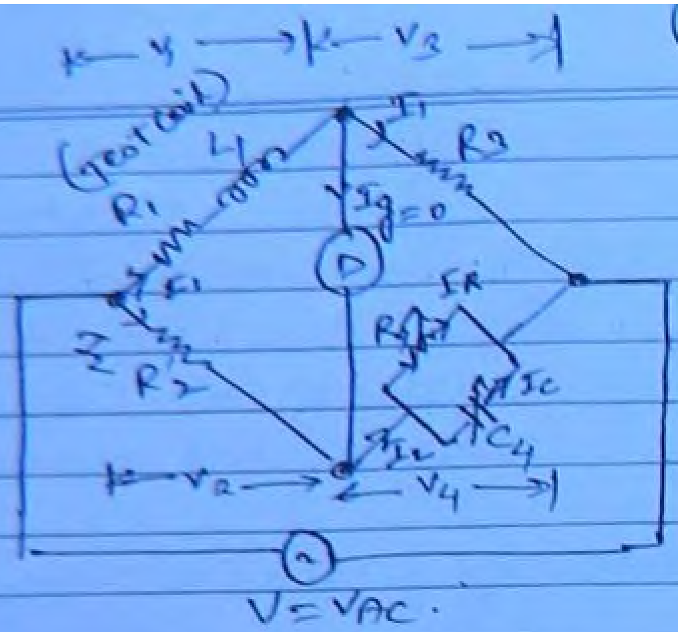
$$Q = \frac{R}{X_L} = \frac{R}{\omega L}$$



$$Q = \frac{R}{X_C} - \omega C R$$

$$Q = \frac{\text{Energy stored by L}}{\text{Energy stored by R}}$$

(i) Max well L-C Bridge - Medium Q Coils ($K < Q < 10$)



(110)

Bridge under balance →

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\frac{R_4}{(1+j\omega C_4 R_4)} = R_2 R_3$$

$$(R_1 + j\omega L_1) R_4 = R_2 R_3 (1 + j\omega C_4 R_4)$$

Seprate Real & Imaginary terms.

$R_1 R_4 = R_2 R_3$

$$\boxed{R_1 = \frac{R_2 R_3}{R_4}}$$

$\omega L_1 R_4 = \omega R_2 R_3 R_4 C_4$

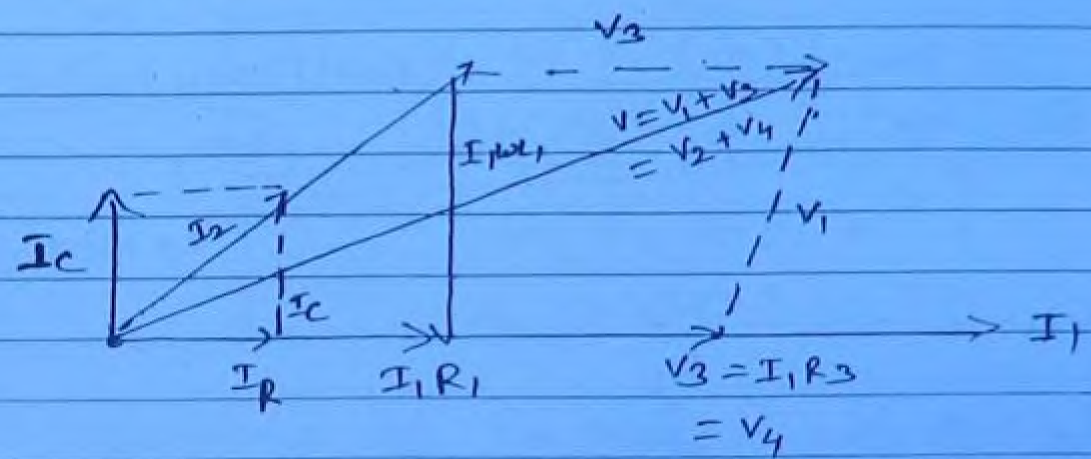
$$\boxed{L_1 = R_2 R_3 C_4}$$

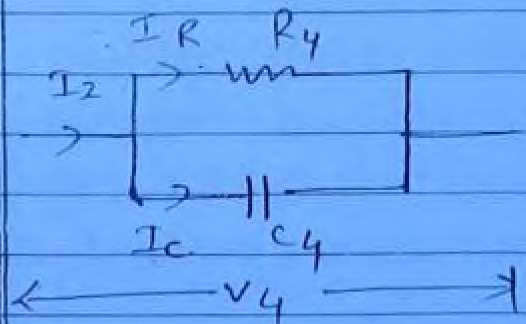
Sliding Balance:-

The equation of R_1 & L_1 the resistance R_2, R_3 are common if these are selected as variable for bridge balanced more no of operation's are required these is called sliding balance. To reduce sliding balance independent parameter's R_4 & C_4 are selated as variables.

Convergance to balance point →
If the variables are lying in the same arm of the bridge than the bridge balance is faster.

Phasor (Balance Condition] :-





$V_4 = I_R R_4$, I_R is in phase with V_4
$= I_C / \omega C_4$, I_C leads V_4 by 90

$$\boxed{I_2 = I_R + I_C}$$

$$Q = \frac{\omega L}{R}$$

$$\boxed{Q = \omega . T}$$

Limitation of Maxwell Bridge →

(1) Not suitable for measurement of high Q coils because the phase angle criteria is not satisfied. Hence Hay's bridge is used.

High Q: $\frac{\omega L_1}{R_1} = \infty$

$\theta_1 + \theta_4 = \theta_2 + \theta_3$

$\theta_4 = \theta_2 + \theta_3 - \theta_1$

$\theta_4 = 0 + 0 - 90^\circ$

$= -90^\circ$

$-\tan^{-1}(\omega C_4 R_4) = -90^\circ$

$\omega C_4 R_4 = \infty$

↳ NOT practical.

$\theta_1 = \tan^{-1}(\omega L_1 / R_1)$

$= \tan^{-1}(\infty)$

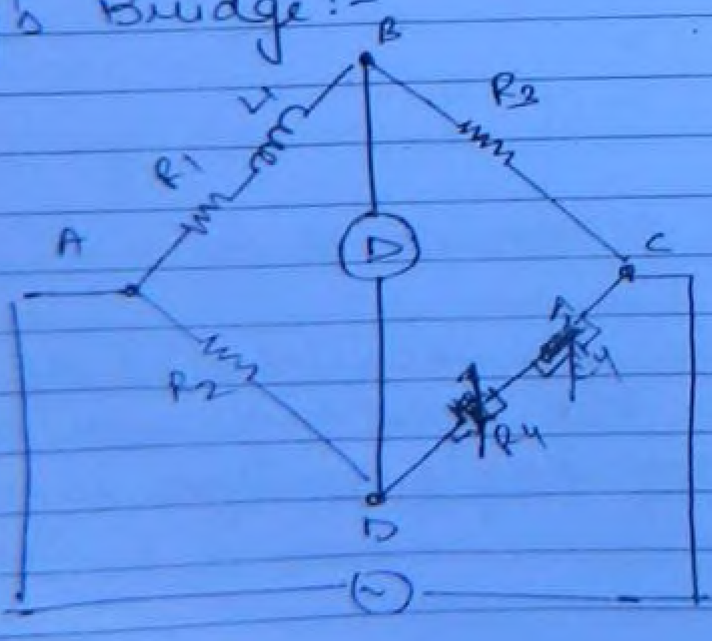
$\theta_1 = 90^\circ$

Low Q:- $\frac{\omega L_1}{R_1} = 0$

$\theta_4 = 0$

In case of low Q coil sliding balance occur so that bridge is not suitable for measurement of low Q coil's. Anderson bridge is used for measurement of low Q coil's.

(2) Hay's Bridge:-



Balance Condition:-

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\left(R_4 + \frac{1}{j\omega C_4}\right) = R_2 R_3$$

Seperate Real & Imaginary terms:-

$$\boxed{L_1 = \frac{R_2 R_3 R_4}{(1+\omega^2 R_4^2 C_4^2)}}$$

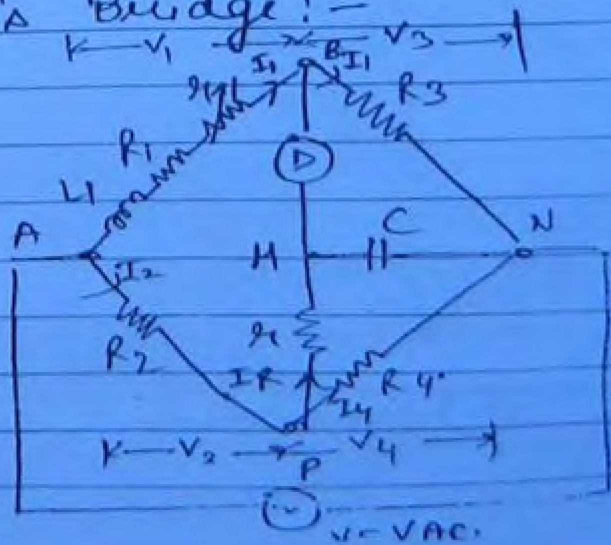
$$\boxed{R_1 = \frac{R_4 R_2 R_3 C_4^2 \omega^2}{1+\omega^2 R_4^2 C_4^2}}$$

The equation's of R_1 and L_1 all the element's of R_2, R_3, R_4 and C_4 are lying so for the bridge balanced intially select R_4 & C_4 as variable's after that select's R_2 and R_3 as variable for bridge balanced.

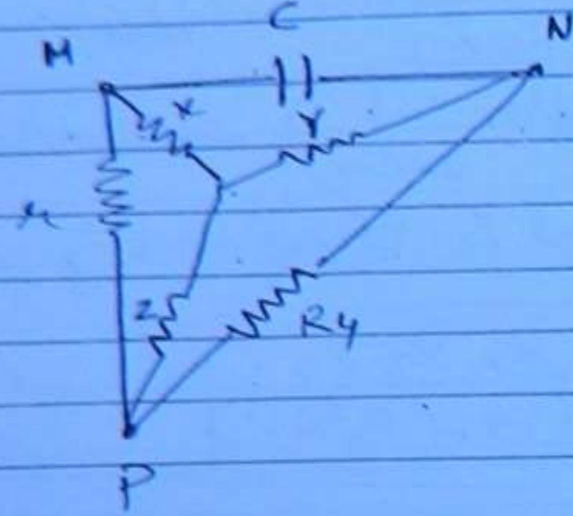
→ Limitation →

Input supply frequency must be known for measurement of R_1 and L_1.

(3) **Anderson's Bridge:-**



Convert Δ into λ

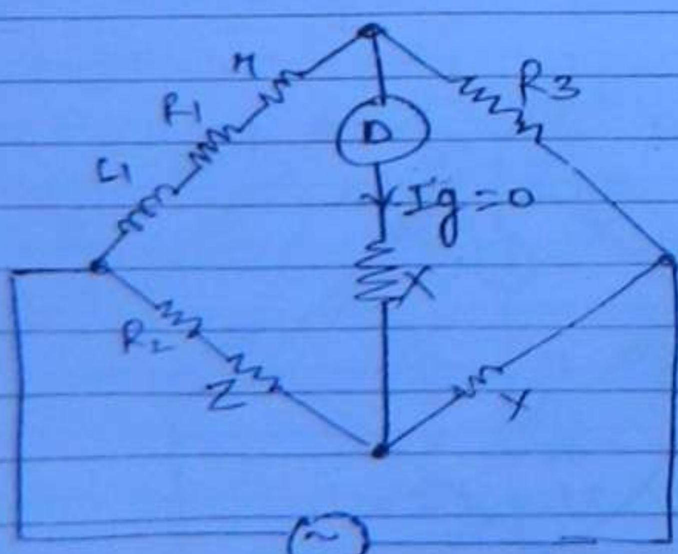


$$Z = \frac{r\,R_4}{r + R_4 + 1/j\omega C}$$

$$Z = \frac{j\omega r C R_4}{j\omega C(r + R_4) + 1} \quad \text{— (1)}$$

$$Y = \frac{R_4 / j\omega C}{R_4 + r + 1/j\omega C}$$

$$Y = \frac{R_4}{1 + j\omega C(R_4 + r)} \quad \text{— (2)}$$



Bridge balance ($I_g = 0$) Ignore x

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + r_1 + j\omega L_1)\, Y = R_3 [R_2 + Z] \quad \text{— } (3)$$

Substitute (1) & (2) in (3) & seperate real & Imaginary terms.

$$\boxed{R_1 = \frac{R_2 R_3}{R_4} - r_1} \; ; \; \boxed{L_1 = \frac{C R_3}{R_4}\left(r(R_1 + R_2) + R_2 R_4 \right)}$$

OBSERVATIONS →

$$V_{AB} = V_{AM}$$

(1) $V_1 = V_2 + r I_R$

(2) $V_{BN} = V_{BM} + V_{mN}$

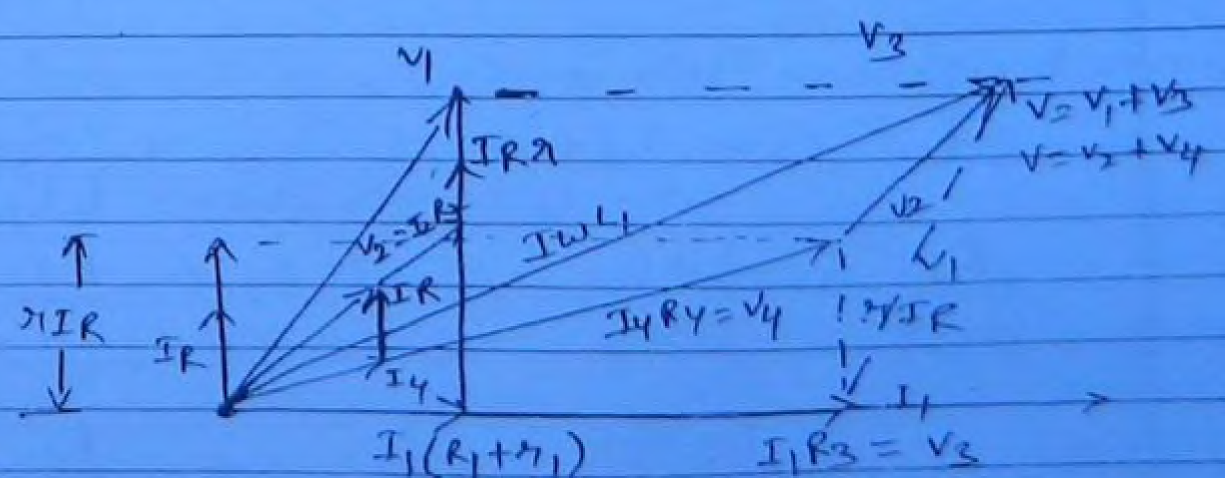
$V_3 = 0 + I_R / \omega C$

I_R leads V_3 by $90°$

(3) $V_{PN} = V_{PM} + V_{MN}$

$V_4 = r I_R + V_3$

(4) $I_2 = I_R + I_4$



(1) I_1 as Ref

(2) $V_1 = I_1(R_1 + r_1) + j\omega L_1$

(3) $V_3 = I_1 R_3$

(4) $V_3 = I_R / \omega C$ I_R leads V_3 by 90°

(5) $I_R r$ in phase to I_R

(6) $V_4 = V_3 + I_R r$

(7) $V_4 = I_4 R_4$

(8) $I_2 = I_4 + I_R$

(9) $V_2 = I_2 R_2$

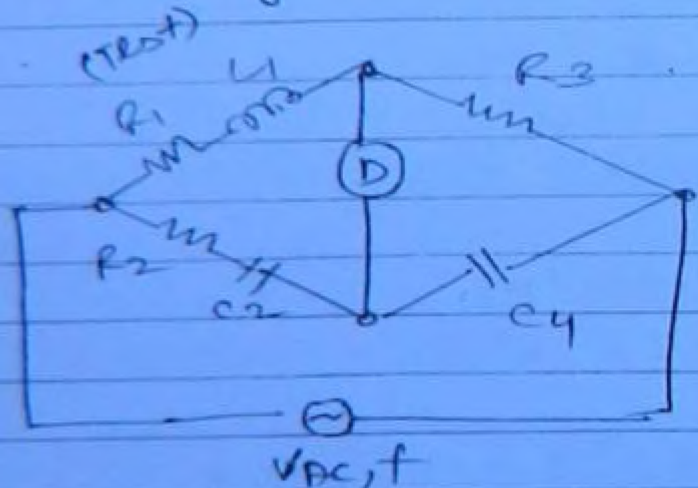
(10) $V_1 = V_2 + I_R r$

(11) $V = V_1 + V_3$

(12) $V = V_2 + V_4$

1/6

Owens Bridge :-



$$Z_1 Z_2 = Z_3 Z_4.$$

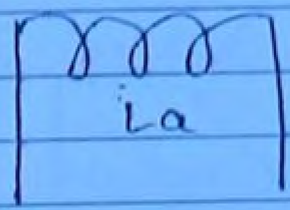
$$(R_1 + j\omega L_1)\left(\frac{1}{j\omega C_4}\right) = R_3\left(R_2 + \frac{1}{j\omega C_2}\right)$$

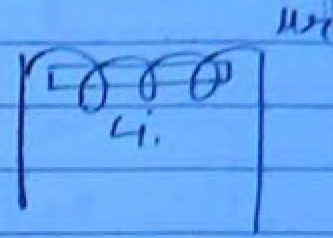
$$\frac{R_1}{j\omega C_4} + \frac{j\omega L_1}{j\omega C_4} = R_3 R_2 + \frac{R_3}{j\omega C_2}$$

$$R_3 R_2 = \frac{L_1}{C_4} \qquad \frac{R_1}{C_4} = \frac{R_3}{C_2}$$

$$\therefore \boxed{L_1 = C_4 R_3 R_2}, \quad \boxed{R_1 = \frac{R_3 C_4}{C_2}}$$

Air Core :- (117) Iron core :-



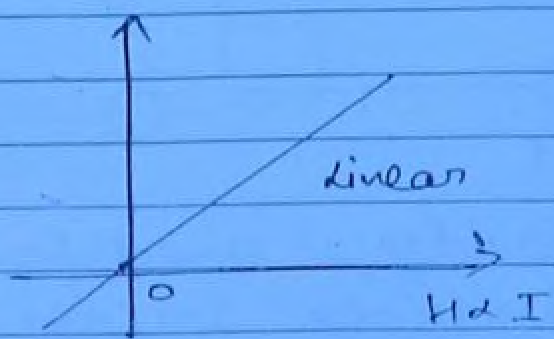


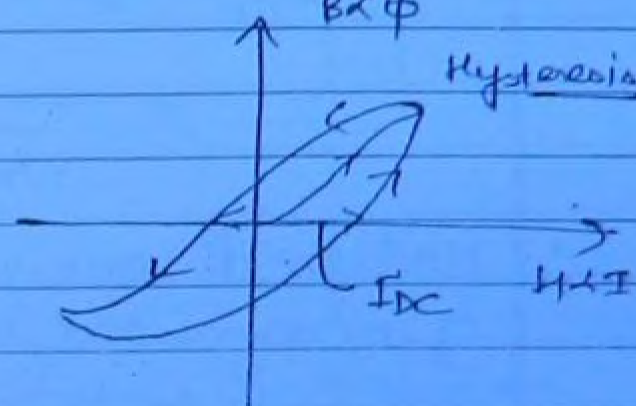
$$L_i = \frac{N^2 \mu_0 A}{l} \cdot \mu_r$$

$$La = \frac{N^2 \mu_0 A}{l}$$

$$\boxed{L_i = La \cdot \mu_r}$$

$B \propto \phi$





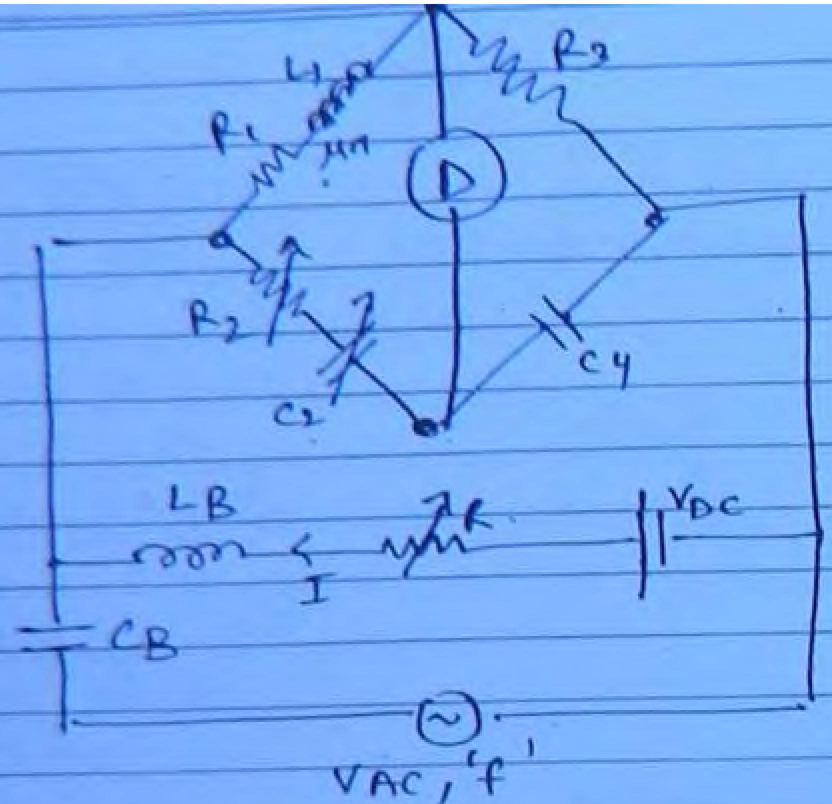
$$L_1 = R_2 R_3 C_4$$

$$\frac{N^2 \mu_0 A}{l} \mu_r = R_2 R_3 C_4$$

$$\boxed{\mu_r = \frac{R_2 R_3 C_4 \cdot l}{N^2 \mu_0 A}}$$

For Eliminating IDC we use modified owen's Bridge.

Modified owens Bridge →



→ Modified owens Bridge is used for measurement of incremental inductance In case of air core coil relation b/w flux and current is linear and there is no residual current present if $\phi = 0$.

→ In case of iron core coil the ϕ and i produces hysteresis effect small DC component is present even $\phi = 0$. Due to this the measured value of inductance does not give accurate value.

→ To compensate DC component external DC source is used in the bridge circuit with a variable resistor. The bridge is once again balanced including dc and ac source and L_1 is calculated in terms of R_2, R_3 & C_4 from this μ_r is calculated

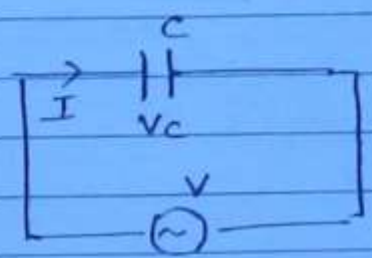
→ The Blocking inductor used to ~~measure~~ block ac and blocking capacitor CB used to block dc.

Measurement of C :-

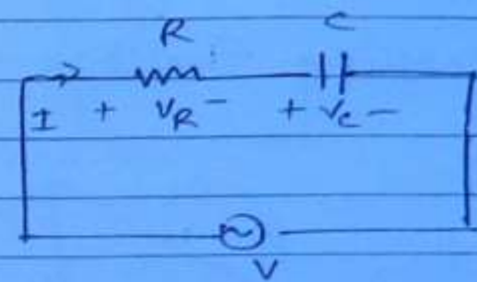
(114)

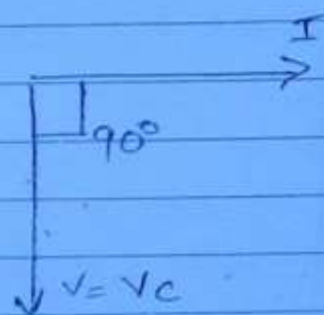
1. De Sauty's bridge — C, D = tan δ
2. Schering bridge — C, D, ϵ_r

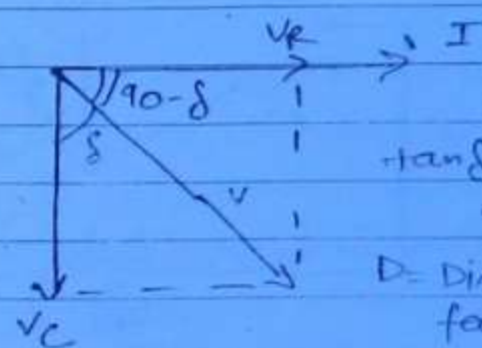
Pure Capacitor



practical Capacitor —







tan δ = loss tangent

D = Dissipation factor.

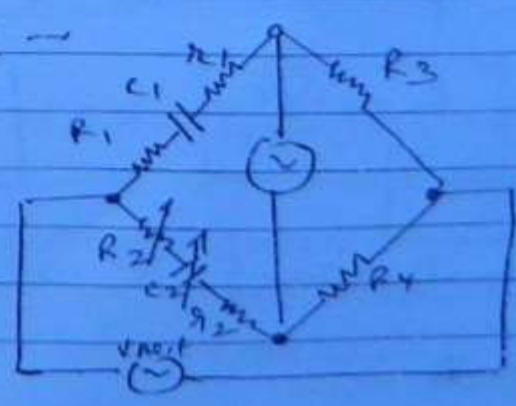
$$D = \tan\delta = \frac{V_R}{V_c} = \frac{RI}{I/\omega C}$$

$$\boxed{D = \omega CR}$$

$$\boxed{D = \frac{1}{Q}}$$

Q = Quality factor.

i) De Sauty Bridge —



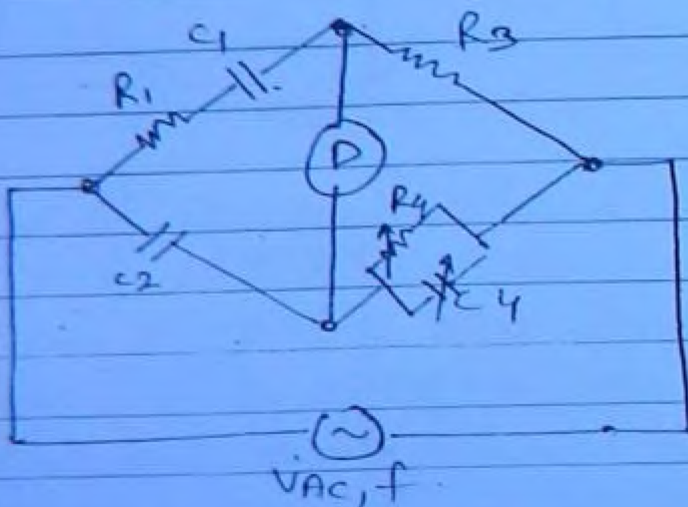
Balance Condition,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\boxed{C_1 = \frac{R_4 C_2}{R_3}} \; ; \quad \boxed{r_1 = \frac{(R_2 + r_2) R_3}{R_1} - R_4}$$

$$\therefore \boxed{D = \omega C_1 r_1} \qquad r_1 = \frac{(R_2 + r_2) R_3}{R_1} - R_1$$

(2) Schering Bridge :-



Bridge Balance :-

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left[\frac{R_4}{1 + j\omega C_4 R_4}\right] = \frac{R_3}{j\omega C_2}$$

Seprate Real & Imaginary terms.

$$\boxed{R_1 = \frac{R_3 C_4}{C_2}} \; ; \quad \boxed{C_1 = \frac{R_4 C_2}{R_3}}$$

$$D = \tan\delta = \omega C_1 R_1$$

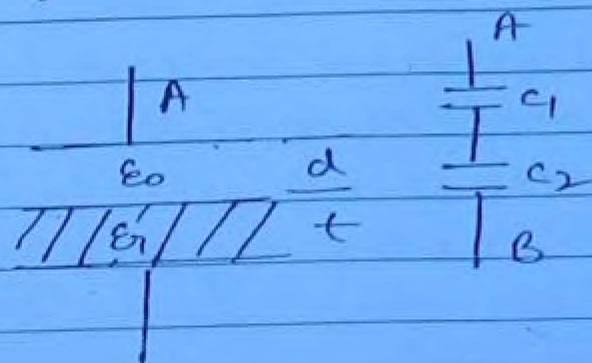
$$\Rightarrow \omega \cdot \frac{\cancel{R_3} C_4}{\cancel{C_2}} \cdot \frac{R_4 \cancel{C_2}}{\cancel{R_3}}$$

$$\boxed{D \Rightarrow \omega R_4 C_4.}$$

(121)

Application →

Including ε_r :-



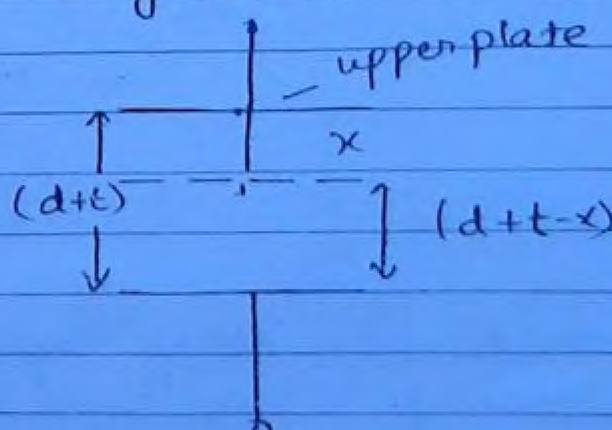
$$C_{AB} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\Rightarrow \frac{\frac{\varepsilon_0 A}{d} \cdot \frac{\varepsilon_0 \varepsilon_r A}{t}}{\frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 \varepsilon_r A}{t}}$$

$$\boxed{C_{AB} = \frac{\varepsilon_0 \varepsilon_r A}{(\varepsilon_r d + t)}} \quad -(1)$$

surement of ε_r :-

Remove dielectric & adjust upper plate to distance 'x' to get same value of C_{AB} :→



$$C_{AB} = \frac{\varepsilon_0 A}{(d+t) - x} = \frac{\varepsilon_0 \varepsilon_r A}{(\varepsilon_r d + t)}$$

$$t + \cancel{\varepsilon_r d} = \cancel{d\varepsilon_r} + t\varepsilon_r - x\varepsilon_r$$

$$\boxed{\varepsilon_r = \frac{t}{t - x}}$$

t = thickness of Dielectric

x = Distance adjusted by upper cap. plate

Due to imbalance in the voltages of the arms of the bridge stray capacitance exist b/w the arms and arms to the ground this will causes error in the measurement of unknown capacitance, inductance, Q, Ms etc. To eliminate this effect a metal earthed screen is provided around the bridge in case of schering bridge and the screen is connected to the earth so that stray capacitance voltage is discharged to the ground and hence its effect is eliminated.

In case of other bridge circuit's Wagnor Earth or Ground Connection is used.

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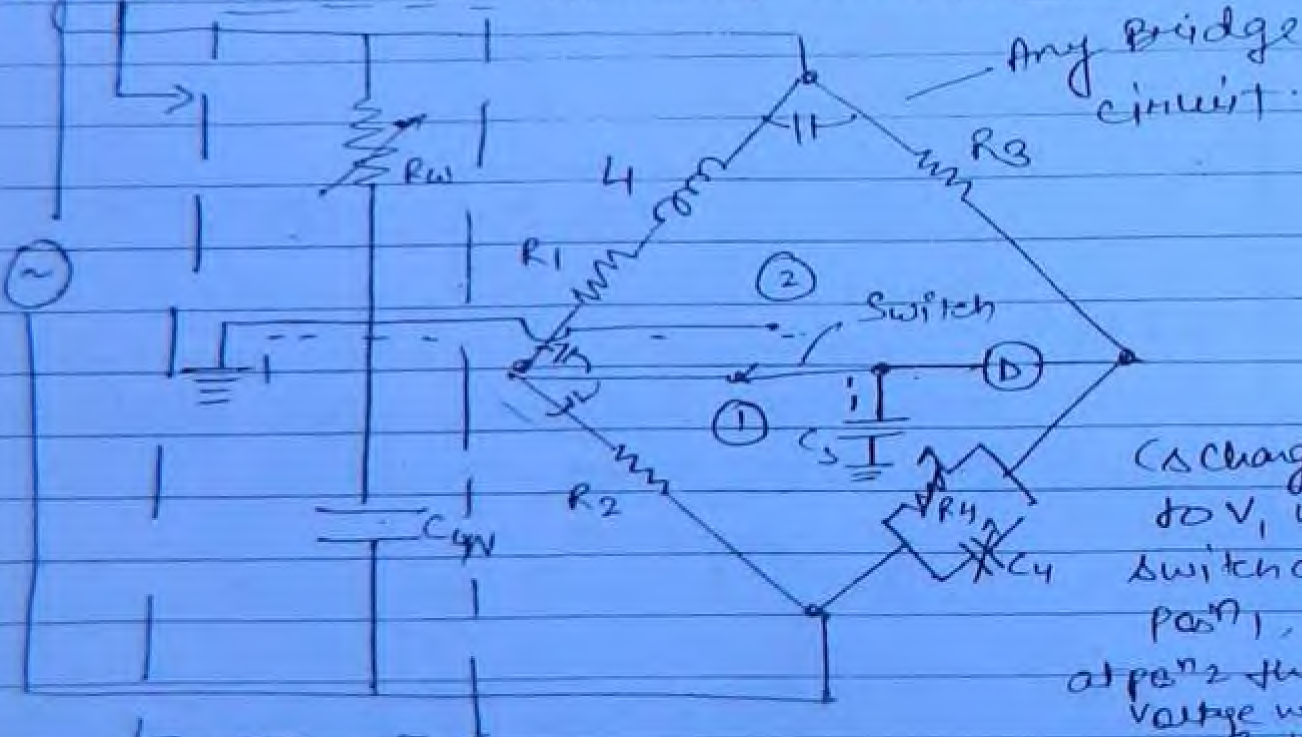
Wagnor Earth or Ground Connection :-



Cs charges to V_1 when switch at posn 1, then at posn 2 the voltage will go to earth

R_W = Wagnor Resistance
C_W = Wagnor Capacitance

C_S = stray capacitance b/w arms & arms to ground

wagner earth or ground connection is used to eliminate effect of stray capacitance existing b/w the bridge arms and arms to the ground.

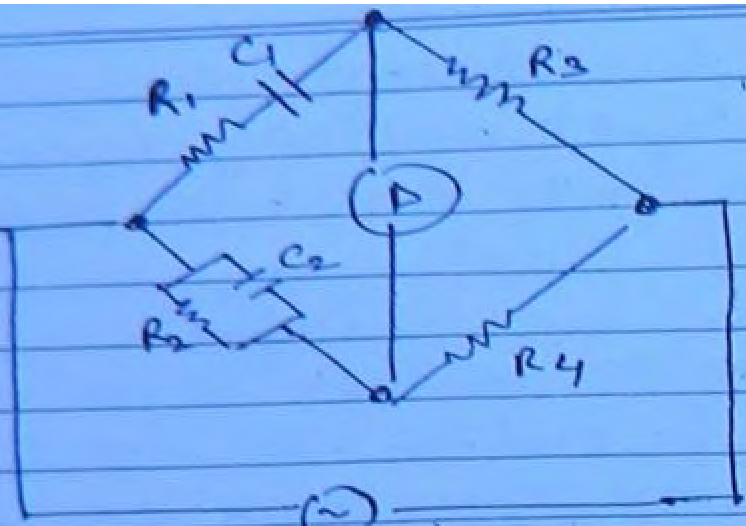
(123)

Procedure:-

Initially, switch is kept at posⁿ 1 by adjusting the resistor R_4 and C_4. The detector is balanced by this time the stray capacitance C_s charge to a voltage V_1.
Now the switch is moved to posⁿ 2 by adjusting R_W bridge is balanced so that the capacitor C_s is discharged to a value to zero through ground.
Now the switch is moved once again to the position 1 by adjusting R_4, C_4 bridge is balanced C_s is charged to voltage V_2 know the switch is moved to posⁿ 2 and this voltage is discharged by R_W. This process continues until in either position of switch 1 or 2. The detector indicate the zero value without changing R_W & R_4, C_4 this corresponding to C_s is at ground potential i.e the effect of stray capacitance is eliminated
If wagner correction is intially appried to the bridge circuit's after balancing C_s (eliminating) the actual values of unknown Qty are measured.

Measurement of frequency (f):-

(W)ein Bridge ($\omega = 2\pi f$)



Test frequency (f).

(124)

Balance of Bridge →

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(R_1 + \frac{1}{j\omega C_1}\right) R_4 = \frac{R_3 \; R_2}{(1 + j\omega C_2 R_2)}$$

$$(1 + j\omega C_1 R_1)\,(1 + j\omega C_2 R_2) R_4 = R_2 R_3 (j\omega C_1)$$

$$\left[1 + j\omega (C_1 R_1 + C_2 R_2) - \omega^2 R_1 R_2 \; C_1 C_2\right] R_4 = j\omega C_1 R_2 R_3$$

Seperate Real & Imag terms.

$$1 - \omega^2 R_1 R_2 C_1 C_2 = 0$$

$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \; ; \; f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Imaginary terms: —

$$\cancel{j\omega}(C_1 R_1 + C_2 R_2) R_4 = \cancel{j\omega} C_1 R_2 R_3$$

$$\frac{\cancel{C_1} R_1}{\cancel{C_1} R_2} + \frac{C_2 \cancel{R}}{C_1 \cancel{R_2}} = \frac{R_3}{R_4}$$

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$$\boxed{\frac{R_1}{R_2} + \frac{C_2}{C_1} = \frac{R_3}{R_4}}$$

→ If $R_1 = R_2 = R$, $C_1 = C_2 = C$

$$\boxed{f = \frac{1}{2\pi C}}, \quad \boxed{\frac{R_3}{R_4} = 2}$$

Limitation →

If a signal containing harmonics than the bridge balanced is difficult. Hence, it is not suitable for measurement of harmonic signal.

Medium R → wheal stone
→ Carey Foster Slidewire → (compar)

Low R → Kelvin Double bridge

High R → Meggar

Measurement of R :-

Winding resistance of Electrical M/c
Earth conductor Resistance

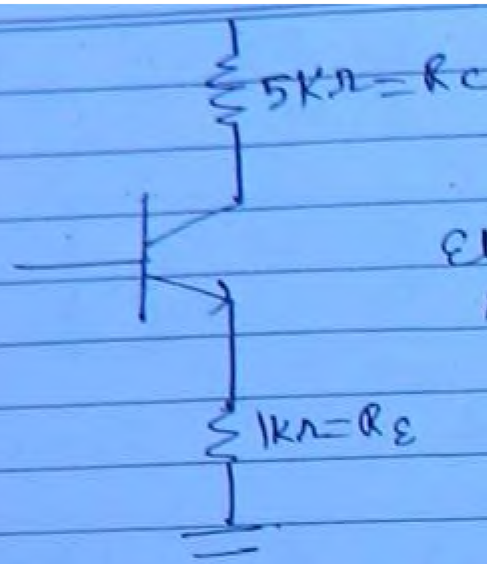
(i) Low Resistance – ($R \leq 1\Omega$)

(ii) Medium Resistance – ($1\Omega < R \leq 100\,k\Omega$)

(iii) High Resistance – $R > 100\,k\Omega$

Electrical Accessories

Winding Insulation of E. M/c, Cable Insulation



Electronic
Accessories.

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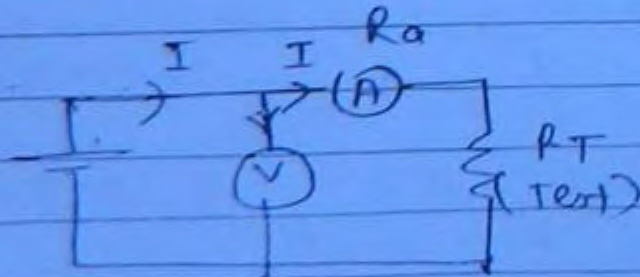
Medium Resistance Methods →

(i) Voltmeter Ammeter Method. (V-I) Method
(ii) wheat Stone Bridge. → Practical.
(iii) Carey Foster Bridge.
(iv) Substitution Method.
(v) Ohm Meter.

(i) V-I Method :-

$$R = \frac{V}{I} = \frac{\text{Voltmeter Reading}}{\text{Ammeter Reading}} = R_m$$

(a) Ammeter near to test Resistance (R_T).



$$V = I(R_a + R_T)$$

$$\frac{V}{I} = R_a + R_T$$

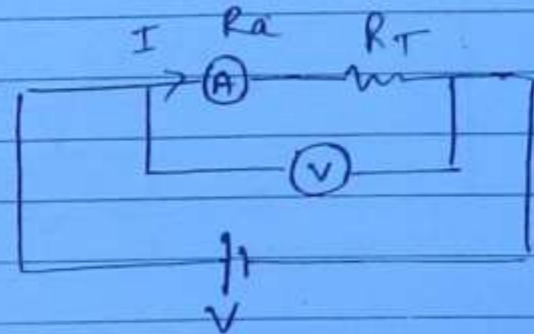
$$\boxed{R_m = R_a + R_T}$$

$$\% \epsilon_r = \frac{R_m - R_T}{R_T} \times 100.$$

$$\boxed{\% \, E_r = \frac{R_a}{R_T} \times 100}$$

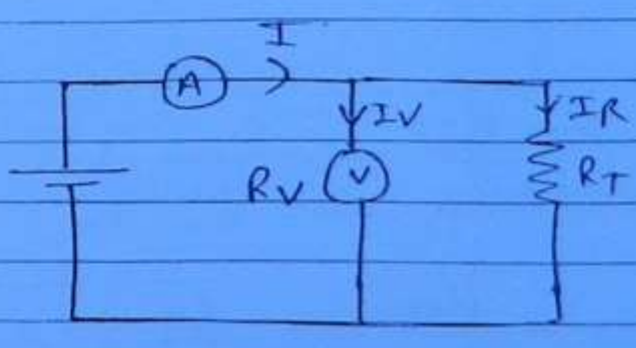
∴ $\boxed{E_r \text{ is low if } R_T >> R_a}$

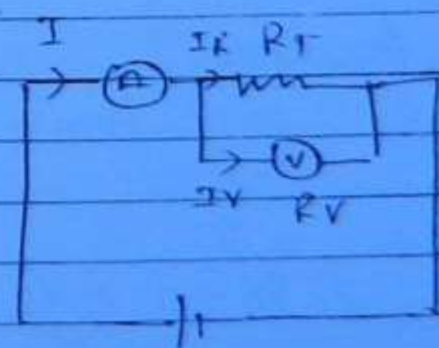
Medium R → 1Ω → Low
→ $100 K\Omega$ → High.



This arrangement is suitable for measurement of R_T Higher than R_a so that error is minimum and hence suitable for measuring high resistances in the medium scale.

(b) Voltmeter near to R_T :-





Assume Ra is negligible.
Internal resistance of voltmeter = Rv.

Exact Analysis →

$I = I_R + I_V$

$I_V = V/R_V$

$I_R = I - I_V$

$I_R = I - V/R_V$

$R_T = V/I_R$

$\% \text{ Error} = \dfrac{R_m - R_T}{R_T}$

$\Rightarrow \dfrac{V/I - V/I_R}{V/I_R}$

$\Rightarrow \dfrac{I_R - I}{I}$

$\Rightarrow -\dfrac{I_V}{I} = -\dfrac{V}{I R_V}$

$$\boxed{\% E_r = -\frac{V}{I R_V} \times 100}$$

V = Voltmeter Reading.
I = Ammeter Reading.

Approximate Analysis →

Assume,

$$R_m = \frac{R_V R_T}{R_V + R_T} \approx R_T = \frac{V}{I}$$

$$\% E_r = \frac{-V}{I R_V} = -\frac{R_m}{R_V} = -\frac{R_T}{R_V}$$

E_r is low if $R_T << R_V$

Hence suitable for low value of R_T.

(ii) Substitution Method :-

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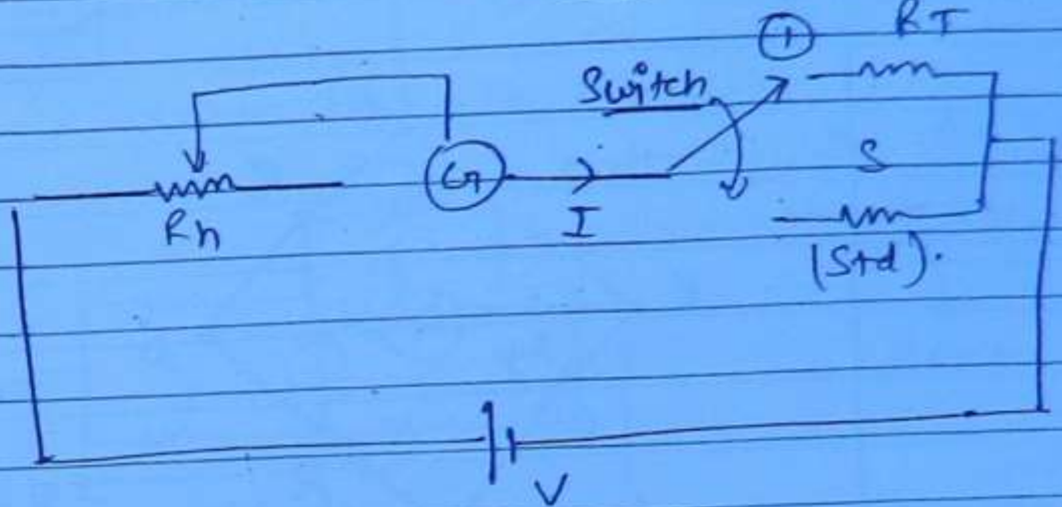


Switch at ①

$$V = I_1 [Rh + R_T + Rg] \quad \text{①}$$

Switch at ②

$$V = I_2 [Rh + Rg + S] \quad \text{②}$$

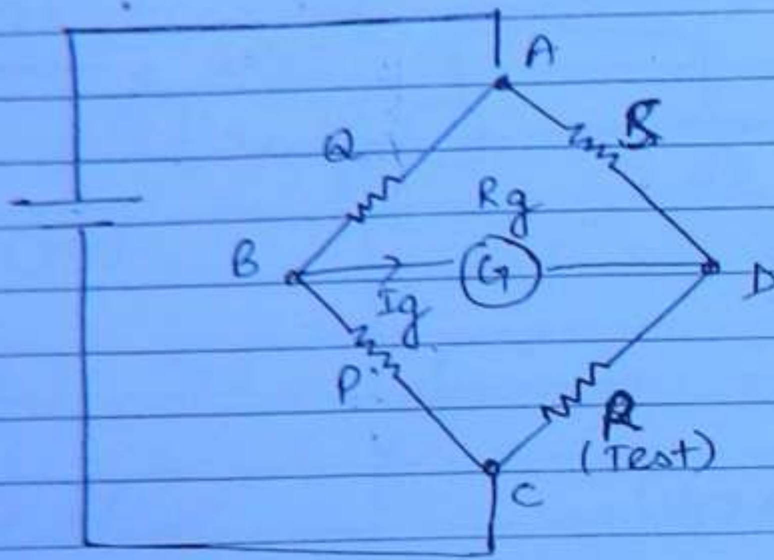
① = ②

$$I_1 [Rh + R_T + Rg] = I_2 [Rh + Rg + S]$$

$$R_T = \frac{I_2}{I_1} [Rh + Rg + S] - (Rh + Rg)$$

If $I_1 = I_2$, $\boxed{R_T = S}$

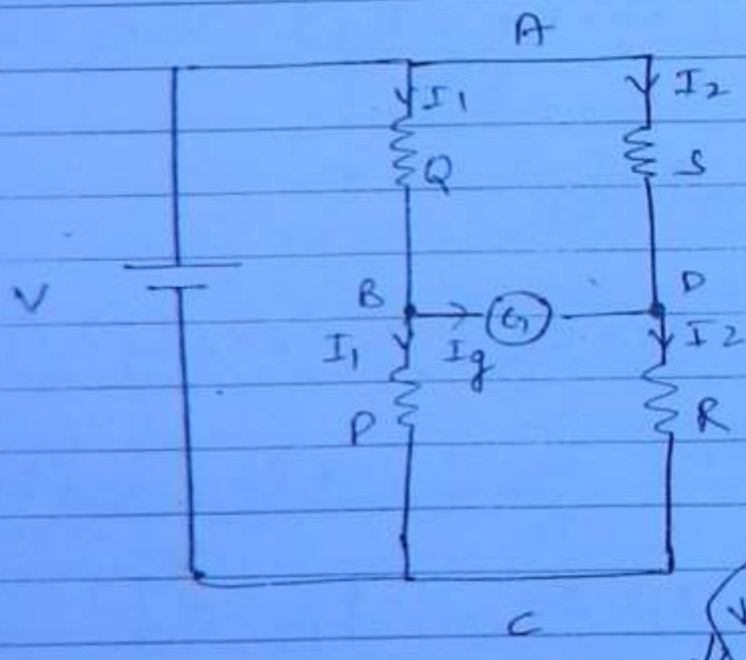
(3) WHEATSTONE BRIDGE :-



$I_g = 0.$

$RQ = PS$

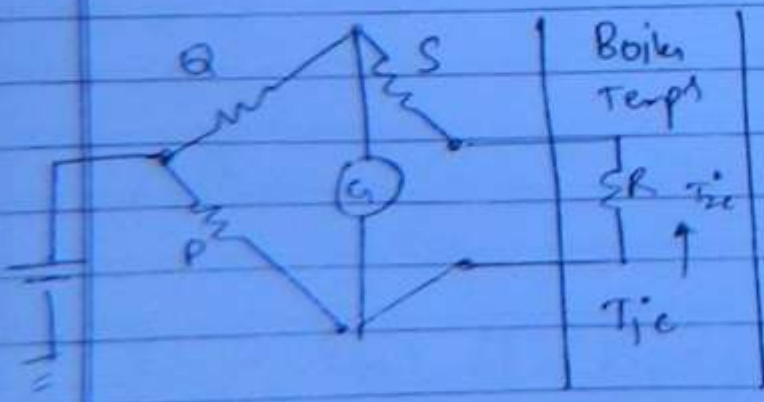
$$\boxed{R = \frac{P}{Q} S}$$



$$V_{dn} = V_B - V_D$$

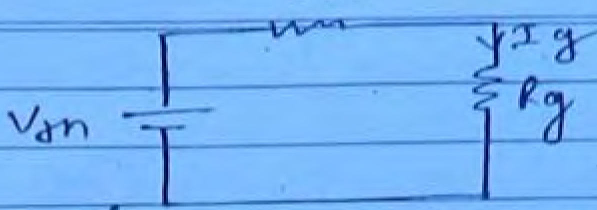
$$\Rightarrow \frac{P}{P+Q} \cdot V - \frac{R}{R+S} \cdot V$$

$$\boxed{V_{dn} = V \left[\frac{PS - RQ}{(P+Q)(R+S)} \right]}$$



$$R_2 = R_1 \left[1 + \alpha (T_2 - T_1) \right]$$

Calculation of Ig :-

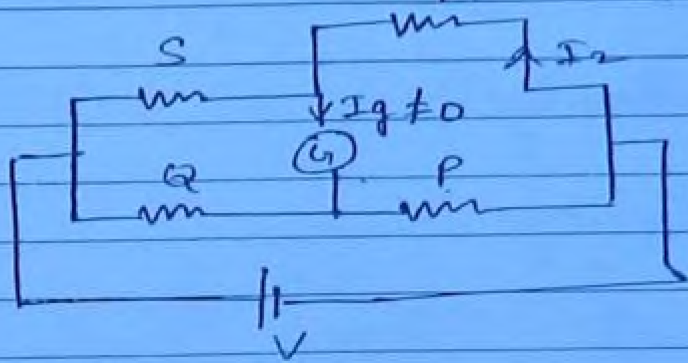


$$\boxed{I_g = \frac{V_{th}}{(R_{th}+R_g)}} \quad (1)$$

(131)

↳ Thevenin Equivalent ckt.

← R ← Fluid   600 T/hour

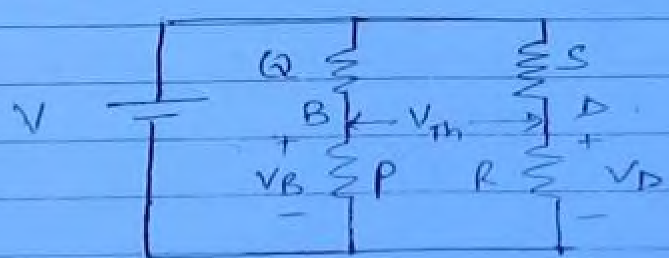


$I_2^2 R = P_L$

↓

Heat.

V_{th} : V_{BD}

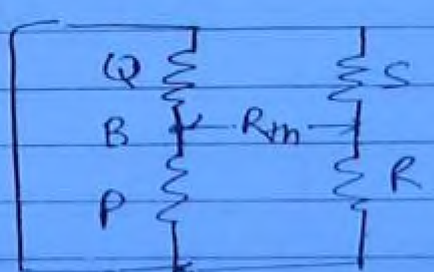


$$V_{th} = V_B - V_D$$

$$= V\left[\frac{P}{P+Q} - \frac{R}{R+S}\right]$$

$$\boxed{V_{th} = V\left[\frac{PS - RQ}{(P+Q)(R+S)}\right]}$$

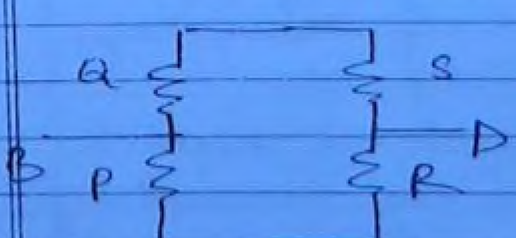
R_{th} : R_{BD} [V = 0]

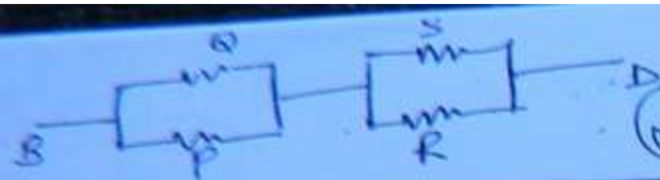


$$R_{th} = R_{BD} = \frac{PQ}{P+Q} + \frac{RS}{R+S}$$

$$I_g = V_{th} / (R_{th} + R_g)$$

$$I_g = \frac{V[PS - RQ]}{(P+Q)(R+S)(R_{th}+R_g)}$$





(1) Bridge is balanced $\boxed{I_g = 0}$

(132)

$$PS - RQ = 0$$

$$\boxed{R = \frac{PS}{Q}} \rightarrow \text{Measurement of Test resistance.}$$

Sensitivities of Bridges :-

(i) Current Sensitivity.

$$S_i = \frac{\theta}{I_g} \quad mm/\mu A.$$

(ii) voltage sensitivity

$$S_V = \frac{\theta}{V_{th}} \quad mm/V$$

(iii) Bridge Sensitivity

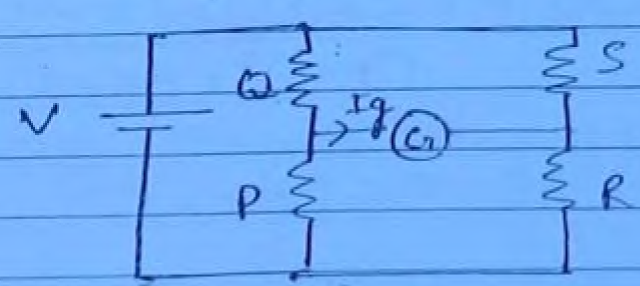
$$S_B = \frac{\theta \rightarrow \text{deflection of galvanometer (mm)}}{(\Delta R/R)} \quad mm$$

$$SB = \frac{V_{th} \cdot S_V}{(\Delta R/R)} \; \dots mm.$$

In case of bridge circuit sensitivity is an important parameter compare to accuracy, resolution, precision, linearity etc for a small change in the non electrical quantity to be measured is to be sensed by changing the resistance R to a value $(R \pm \Delta R)$

This change should be detected by change in the current through the galvanometer (I_g) which produces deflection so that the galvanometer indicates max^m value i.e its sensitivity is maximum. 133

Condition for Max^m Sensitivity [S_{Bmax}]

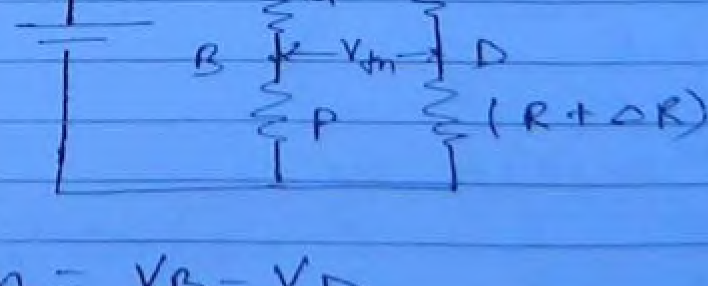


Under Bridge balance:-

$$\frac{P}{Q} = \frac{R}{S}$$

$$\boxed{\frac{P}{P+Q} = \frac{R}{R+S}} \quad \text{—— (1)}$$

for small change in R to $(R+\Delta R)$:-



$$V_{th} = V_B - V_D$$

$$\Rightarrow V\left[\frac{P}{P+Q} - \frac{(R+\Delta R)}{(R+\Delta R+S)}\right]$$

(134)

$$\Rightarrow V\left[\frac{R}{R+S} - \frac{R+\Delta R}{(R+\Delta R+S)}\right]$$

From I^{st}.

$$V_{th} = V\left[\frac{R(R+\Delta R+S)-(R+S)(R+\Delta R)}{(R+S)^2+(R+S)\Delta R}\right]$$

Assume $(R+S)\Delta R << (R+S)^2$

$$V_{th} = V\left[\frac{\cancel{R^2} + R\cancel{\Delta R} + RS - \cancel{R^2} - R\cancel{\Delta R} - S\Delta R}{(R+S)^2}\right]$$

$$\boxed{|V_{th}| = \frac{V \cdot S \cdot \Delta R}{(R+S)^2}} \quad -(2)$$

$$S_B = \frac{V_{th} \cdot S_V}{(\Delta R/R)} = \frac{VS\,\Delta R \cdot S_V}{(\cancel{\Delta R}/R)(R+S)^2}$$

$$\Rightarrow \frac{V S_V}{\left(\frac{R^2+2RS+S^2}{RS}\right)}$$

$$S_B = \frac{V S_V}{\left(\frac{R}{S}+\frac{S}{R}+2\right)}$$

for Max^m value of S_B

$$\boxed{\frac{R}{S} = \frac{S}{R} = \frac{P}{Q} = 1} \rightarrow \text{condition for } S_B\,max$$

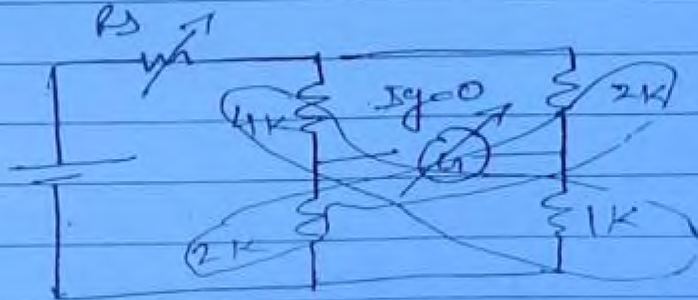
$$\boxed{S_{B(max)} = \frac{V S_v}{4}}$$

Under Bridge balance condition.

(i) The product of resistances in opposite arms of the bridges are equal i.e $PS = RQ$.

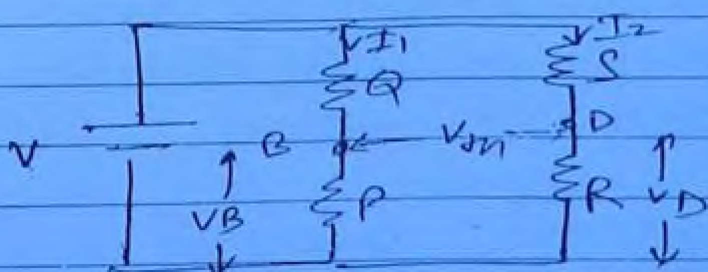
(ii) If $PS = RQ$ than $I_g = 0$ and it is independent of galvanometer internal resistance and source voltage internal resistance.

(iii) The voltage across the galvanometer is zero.



when $2K \times 2K = 4K \times 1K$ than

$I_g = 0$



$PS = RQ$

$V_{th} = V_{BD} = V_B - V_D$ 0

$V_B = V_D$ $\boxed{I_g = 0}$

$I_1 P = I_2 R$

$\frac{I_1}{I_2} = \frac{R}{P}$

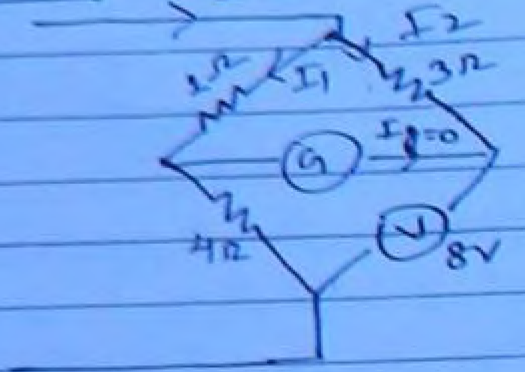
$V_B = V_D$

$I_1 Q = I_2 S$

$I_1/I_2 = S/Q$.

$$\boxed{\frac{R}{P} = \frac{S}{Q}}$$

Find $\underline{I}$:-



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$4I_1 = 8$	$2I_1 = 3I_2$
$I_1 = 2\,A$	$2 \times 2 = 3I_2$
	$I_2 = 4/3\,A$

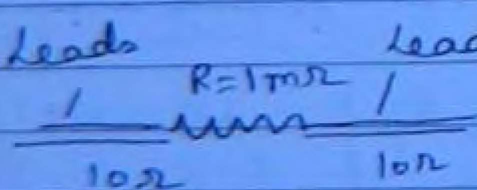
$I = I_1 + I_2$

$\Rightarrow 2 + 4/3$

$\boxed{I \Rightarrow 10/3\,A}$

Limitation of wheat Stone Bridge →

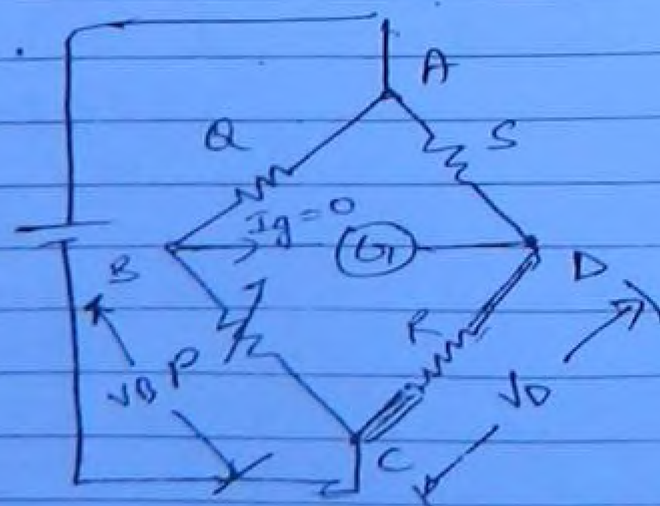
1) Not suitable for measurement of low resistance because it cannot eliminate the effect of lead resistance.

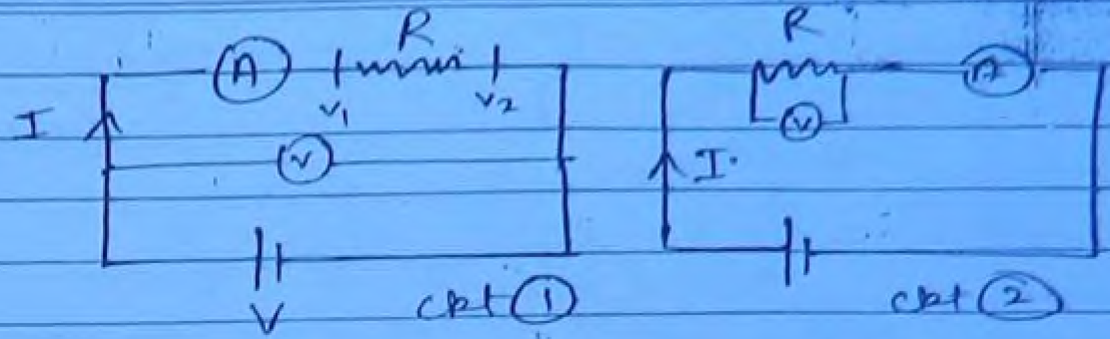


$R = \dfrac{P}{Q} S$

$\Rightarrow 20.001\,\Omega$

$\approx 20\,\Omega$





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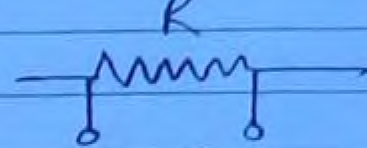
Symbol:-

Low Resistance :- 

Medium Resistance :- 

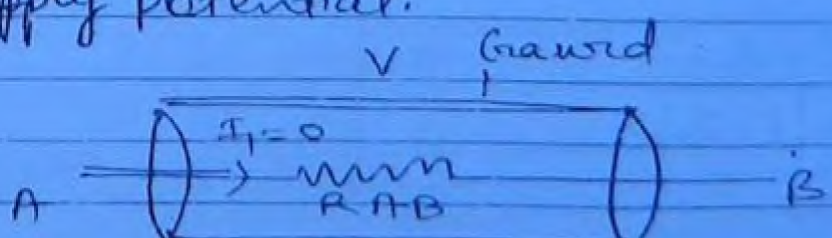
High Resistance :- 

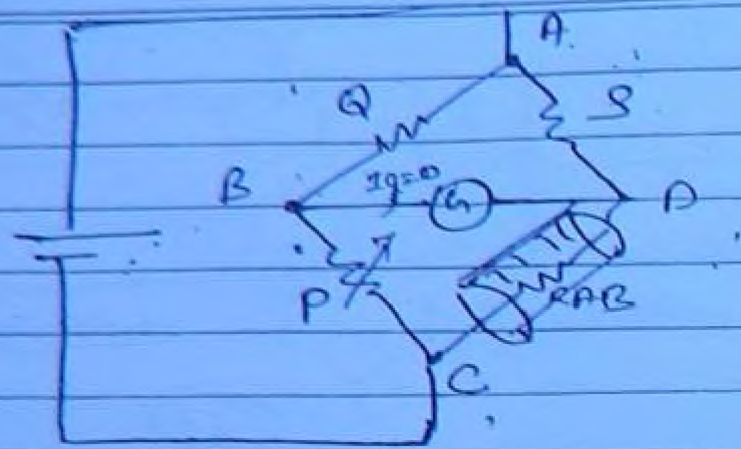
G = Gaurd terminal.

→ Kelwin Double bridge is used for limi elimination of lead resistance hence used for measurement of low resistance.

→ Not suitable for measurement of high resistance due to sensitivity of the bridge is affected due to the leakage current's flowing in the high resistance material.

→ To eliminate the leakage current affect's guard terminal is provided which is maintained at supply potential.





Mego ohm bridge.

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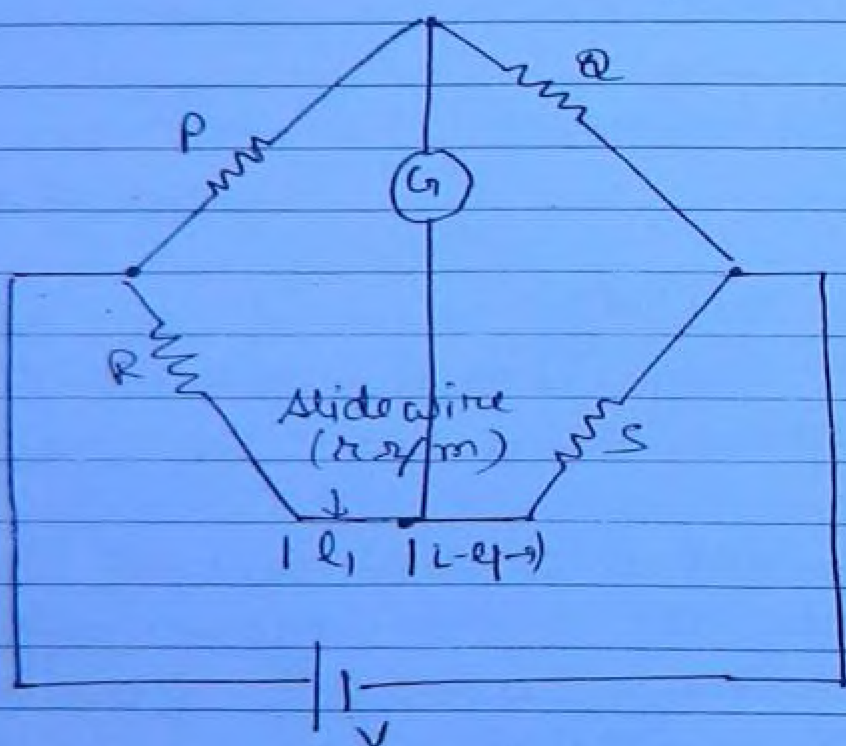
4) Carey Foster slide wire →



Bridge balance at A :-

$$[R + l_1 r]\,Q = P\,[S + (L - l_1) r]$$

$$\boxed{\frac{R + l_1 r}{S + (L - l_1) r} = \frac{P}{Q}} \quad \text{— (1)}$$

With interchanging P & S →

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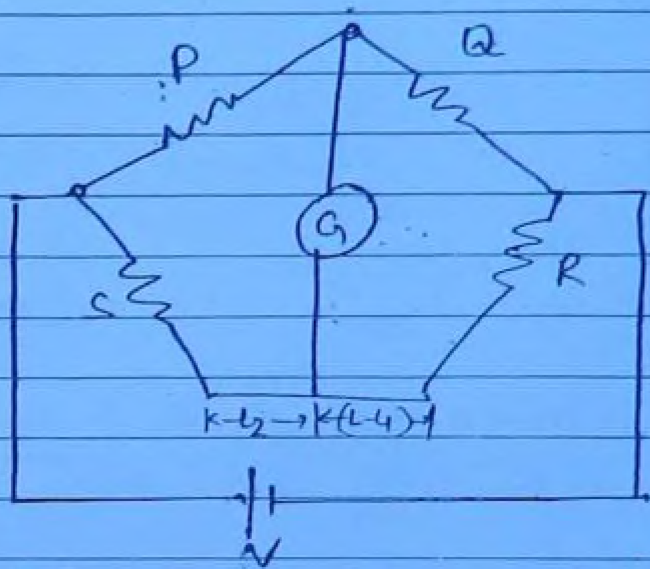

Bridge balanced at B :-

$$\frac{P}{Q} = \frac{S + l_2 r}{R + (L - l_2) r} \quad -(2)$$

$$(1) = (2)$$

Solve,

$$\boxed{R - S = (l_2 - l_1) r}$$

Unknown resistance R is measured by comparining with standard resistance value

Ohm Meter :-

$$R = V / I$$

$V \Rightarrow$ Constant.

$$R \propto 1/I$$

$$R \propto \frac{1}{\text{Ammeter Reading}}$$

(1) Series ohm Meter:-

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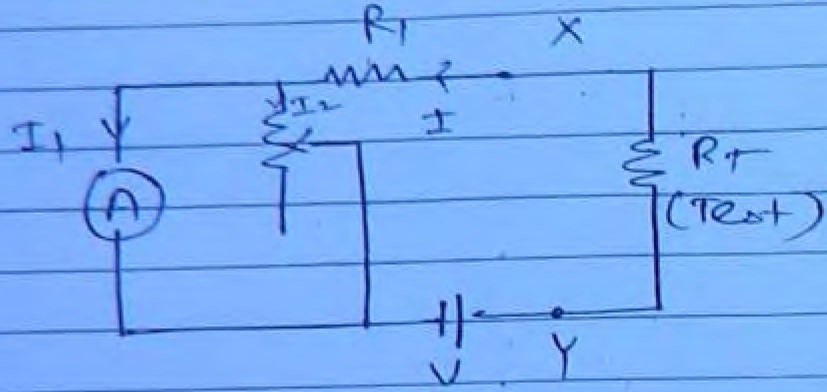
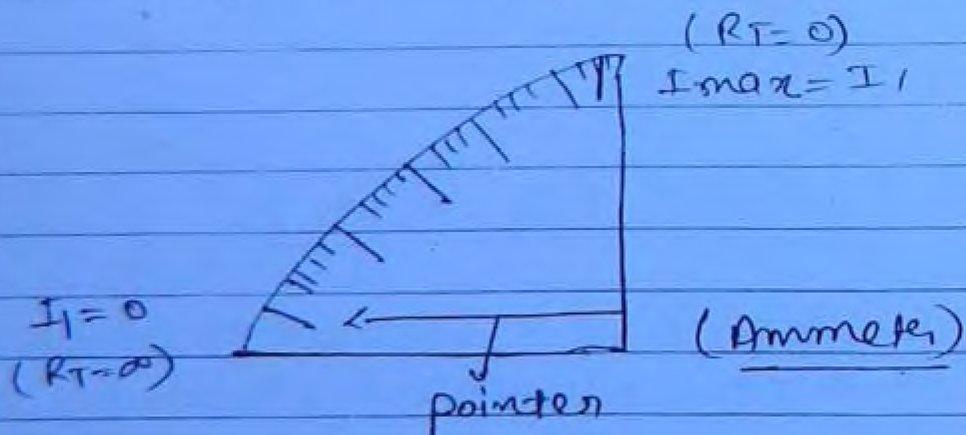


R_2 = Zero Resistance adjustment Resistor.
R_1 = Current Limiting Resistor.

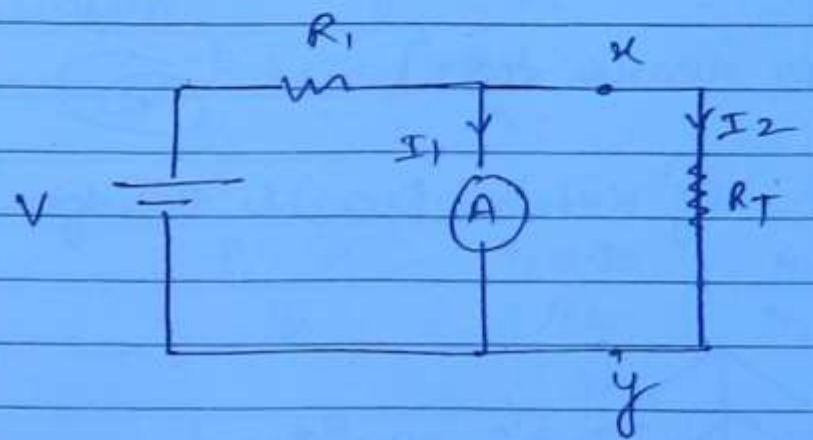
X-Y open	X-Y short.
$R_T = \infty$	$R_T = 0$
$I_1 = 0$	$I_1 = max$



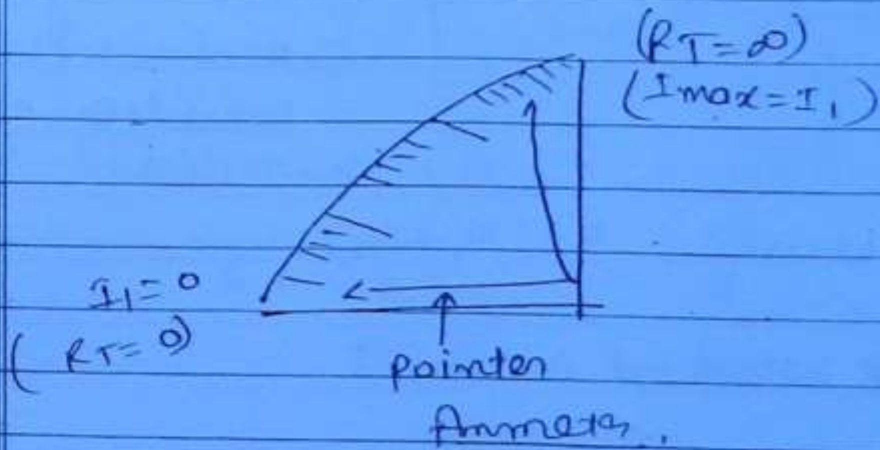
The ammeter is Caliberated to measure the resistance over wide range from low value to the high value.
Ammeter is connected in series to the test resistance hence it is called series ohm meter. If the battery source potential is reduced than the current flowing through meter is reduces and indicate less value

at $R_T = 0$ for adjustment of this value the variable resistance R_2 is used.

(ii) Shunt ohm Meter:- (T4)



x-y open	x-y short
$R_T = \infty$	$R_T = 0$
$I_1 = I_{max}$	$I_1 = 0$



→ It is used for measurement of low resistance in the medium scale.

Low Resistance Measurement Methods:-

(i) Kelvin Double bridge. → Most practical (Accuracy is more).
(ii) potentiometer Method.
(iii) V-I Method
[voltmeter nearer to R_T]

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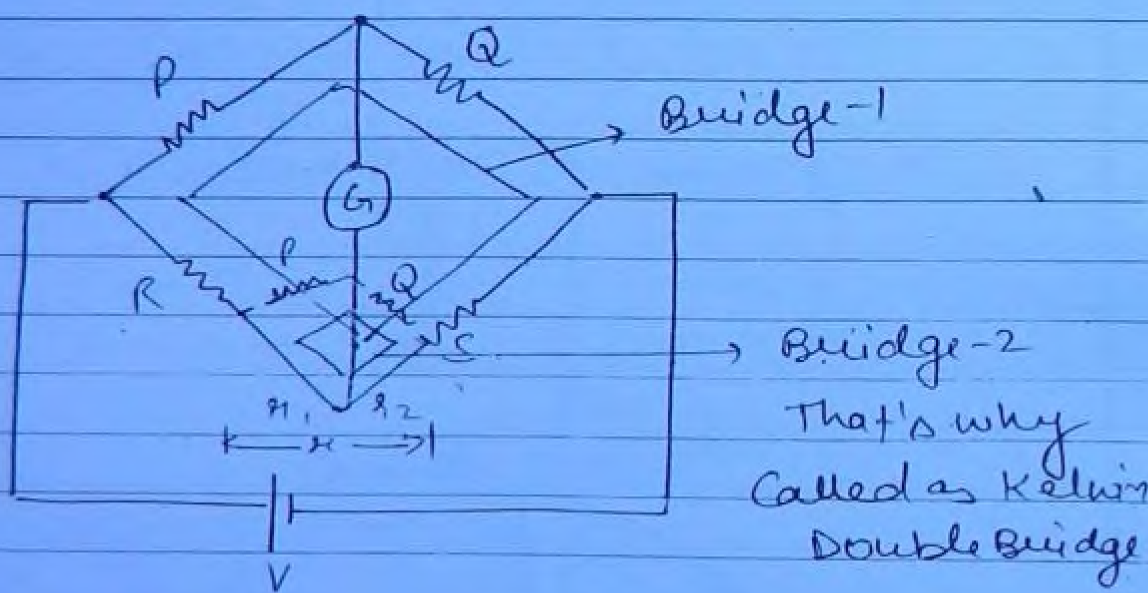
Basic Circuit of Kelvin Double Bridge:-



Lead resistance $r = (r_1 + r_2)$

Bridge ~~at~~ balanced at A :-

$$\frac{P}{Q} = \frac{R + r_1}{S + r_2}$$

$$\boxed{(R + r_1) = \frac{P}{Q}(S + r_2)} \quad \text{— (1)}$$

Including lead Required → $\boxed{R = \frac{P}{Q} \cdot S}$

Select, $\dfrac{P}{Q} = \dfrac{p}{q} = \dfrac{r_1}{r_2}$

$$\frac{p}{q} = \frac{r_1}{r_2}$$

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$$\frac{p}{q} + 1 = \frac{r_1}{r_2} + 1$$

$$\frac{p+q}{q} = \frac{r_1 + r_2}{r_2} = \frac{r}{r_2}$$

$$r_2 = \frac{qr}{(p+q)} \quad - (2)$$

$$r_1 = r - r_2 = r - \frac{qr}{p+q}$$

$$\boxed{r_1 = \frac{pr}{p+q}} \quad - (3)$$

Substitute (2), (3) in (1)

$$R + \frac{pr}{p+q} = \frac{P}{Q} S + \frac{P}{Q} \cdot \frac{qr}{(p+q)}$$

Put $P/Q = p/q$

$$R + \cancel{\frac{pr}{p+q}} = \frac{P}{Q} \cdot S + \cancel{\frac{p}{q} \frac{qr}{(p+q)}}$$

$$\boxed{R = \frac{P}{Q} S} \rightarrow \text{No Load Resistance}$$

Practical Circuit of Kelvin Double bridge →

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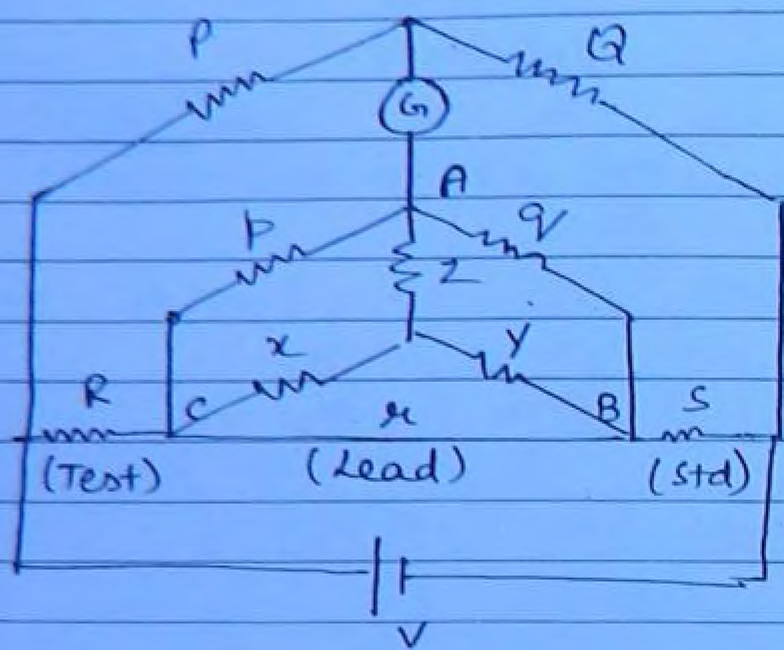
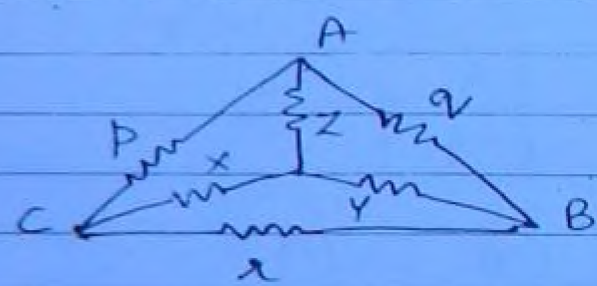
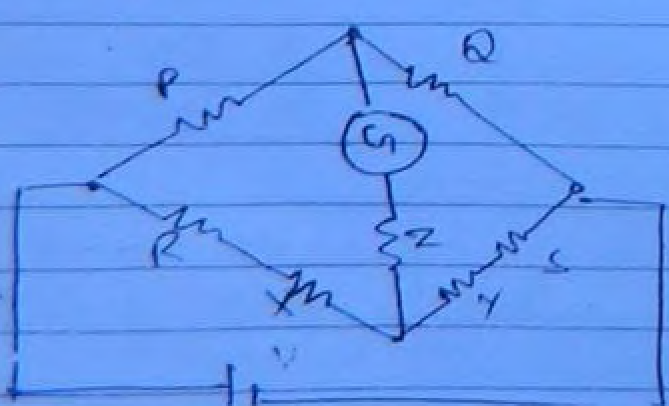


Convert Δ into Y



$$x = \frac{pr}{(p+q+r)} \quad \text{—— (1)}$$

$$Y = \frac{qr}{(p+q+r)} \quad \text{—— (2)}$$



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Bridge under balance (Ignore r)

$$\frac{P}{Q} = \frac{R+X}{S+Y}$$

$$R+X = \frac{P}{Q}(S+Y) \quad -(3)$$

Substitute (1) & (2) in (3) and solve.

$$\boxed{R = \frac{P}{Q}S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]}$$

Including Lead resistance Effect.

→ If $P/Q = p/q$

$$\boxed{R = P/Q \cdot S}$$

No Lead Resistance Effect.

P, Q = Outer ratio arms.
p, q = Inner ratio arms.

→ The effect of lead resistance is eliminated if $P/Q = p/q$. And Hence, this is suitable for measurement of low resistance. And the accuracy of instrument is more.

Application →

Used for measurement of winding resistance of electrical motor, generator's and transformer's and also used for

measurement of earth conductor resistance

Potentiometer Method :-

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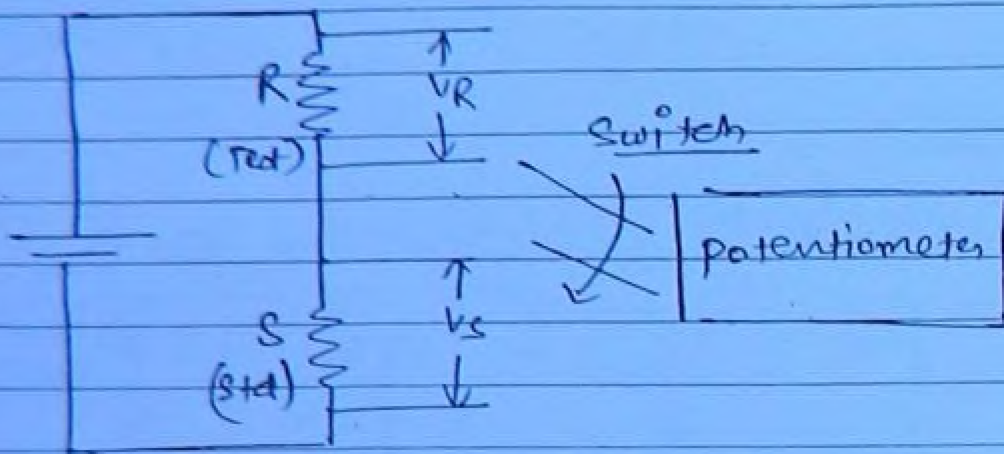


Switch across R

$V_R = IR$

$$\boxed{I = V_R/R}$$

Switch across S

$V_S = I \cdot S$

$$\boxed{I = V_S/S}$$

$$V_R/R = V_S/S$$

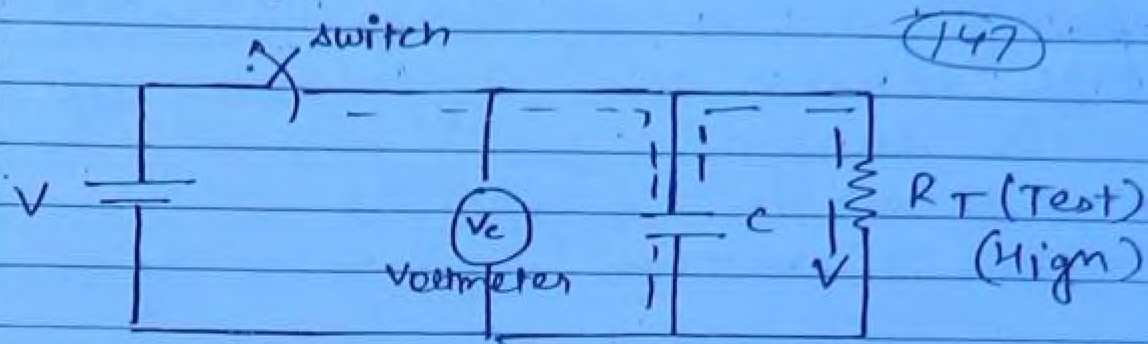
$$\boxed{R = S \cdot V_R/V_S}$$

High Resistance Measurement Methods →

1. Meggar → Most practical
2. Loss of charge method
3. Direct Deflection method
4. Mega ohm bridge.

Loss of charge Method :-

(147)



Switch opened at $t = 0$

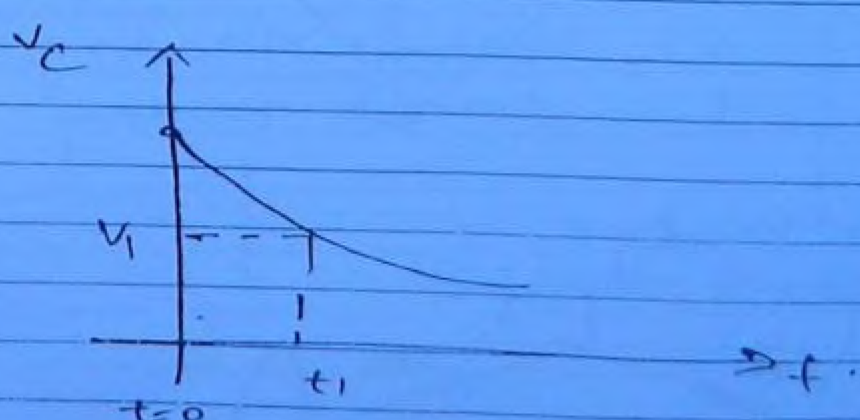
$$V_c = V \cdot e^{-t/RC}$$

$$\frac{V}{V_c} = e^{t/RC}$$

$$\frac{t}{RC} = \ln_e (V/V_c)$$

$$R = \frac{t}{C \ln_e (V/V_c)}$$

$$\boxed{R = \frac{0.4343\, t}{C \log_{10} (V/V_c)}}$$; where $t =$ seconds.



Q Cable insulation resistance is measured by using a loss of charge method the voltage across the capacitance of 0.4343 μF is dropped from 10 V to 1 V in 2 min's when the switch is opened at time $t=0$ than the unknown resistance of the cable is

a) $1\,M\Omega$ c) $120\,M\Omega$ 148

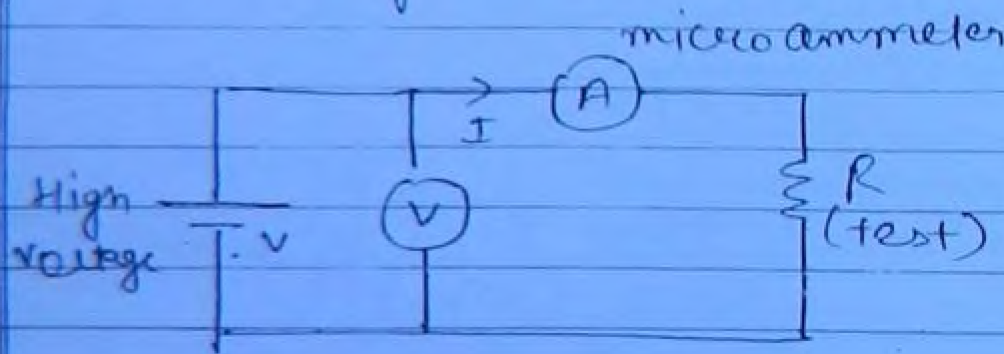
b) $2\,M\Omega$ d) $180\,M\Omega$

Ans

$$R = \frac{0.4343 \times 2 \times 60}{0.4343 \times 10^{-6} \log(10/1)}$$

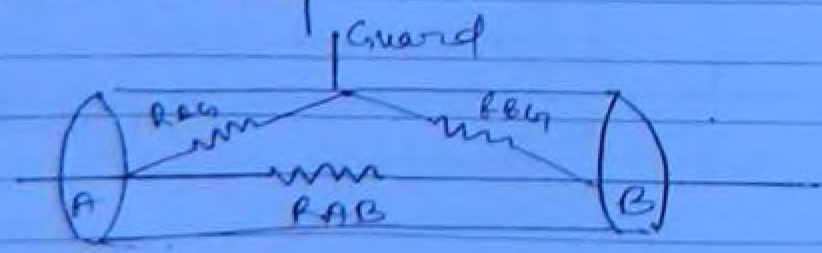
$$\Rightarrow 120\,M\Omega$$

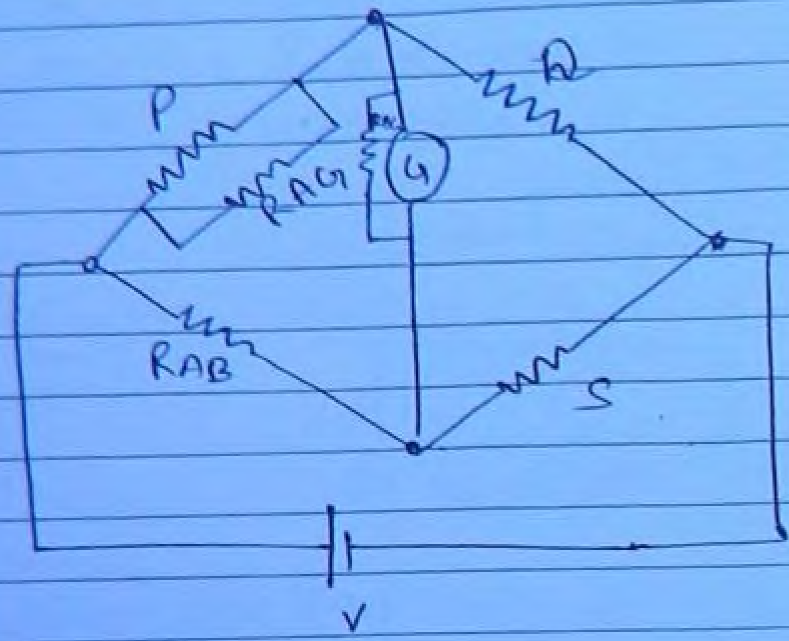
Direct Deflection Method :-



$$R = \frac{V}{I} = \frac{\text{Voltmeter Reading}}{\text{Ammeter Reading}}$$

Mega ohm Bridge :-





Mego Ω bridge is working on a principle of wheatstone bridge used to measure high value of resistance's by connecting guard terminal to the supply potential so that the effect of leakage ~~terminals~~ current are eliminated.

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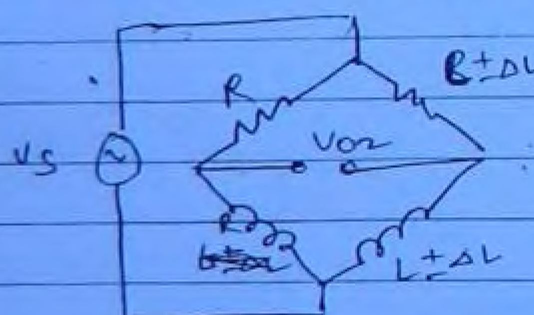


① $\frac{\Delta L}{L} = 0.1$

② $R = \omega L$

$$V_{th} = V_{o1} = V_B - V_D$$

$$= V_s \left[\frac{R}{R+R} - \frac{j\omega(L \mp \Delta L)}{j\omega(L \pm \cancel{\Delta L} + L \mp \cancel{\Delta L})} \right]$$

$$\Rightarrow V_s \left(\frac{1}{2} - \frac{(L \mp \Delta L)}{2L} \right)$$

$$V_o = \frac{V_s}{2} \left(1 - \left(\frac{L}{L} + \frac{\Delta L}{L} \right) \right] \text{ or } \frac{V_s}{2} \left[1 - \left(\frac{L}{L} - \frac{\Delta L}{L} \right) \right]$$

$$\frac{V_S}{2}\left[-0.1\right] \quad \text{or} \quad \frac{V_S}{2}\left[0.1\right]$$

$$|V_{o1}| = 0.05\, V_S \quad (15)$$

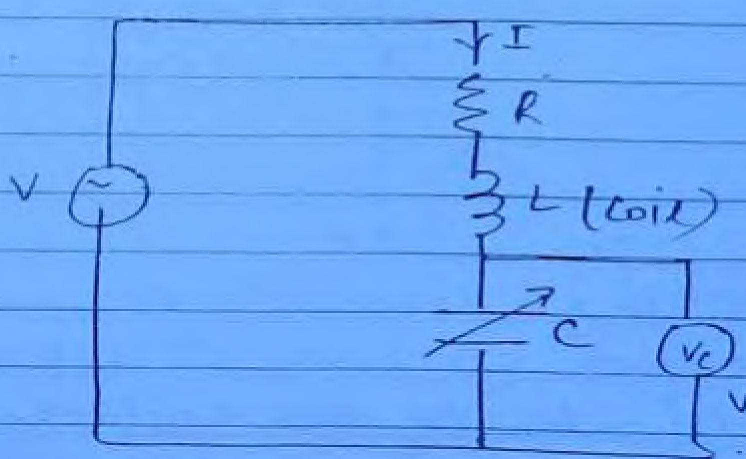
similarly

$$\boxed{V_{o2} = .05\, V_S}$$

Q Meter Or Voltage Magnifier →

Q Meter is working on a principle of series resonance.

Used for measurement of quality factor of the coil, unknown capacitance, inductance self or distributed capacitance of the coil etc.



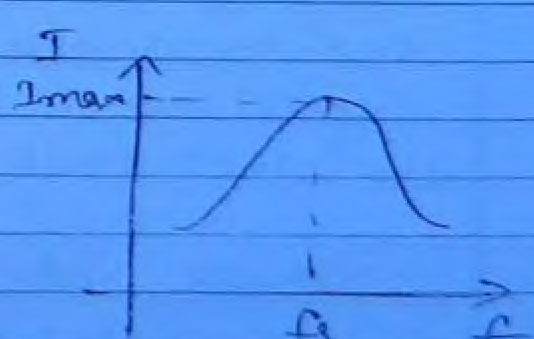
$$Q = \frac{X_C}{R} = \frac{X_L}{R}$$

Series resonance. $X_L = X_C$

$$I = V/R$$

$$V_C = I X_C$$

$$\Rightarrow V \cdot X_C / R$$



$$\boxed{V_C = V \cdot Q}$$

$$\boxed{V_C \propto Q} \quad V = \text{constant}$$

~~Practical Qmeter~~

A voltmeter is connected across the variable capacitor this meter is calibrated to measure the Quality factor of the coil under test.

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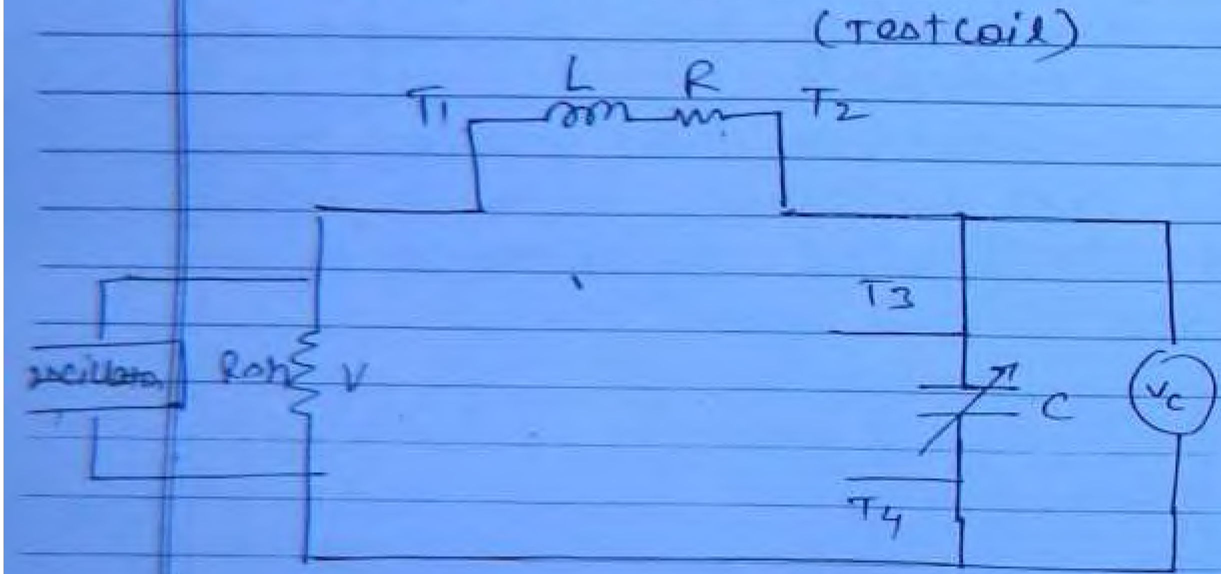
Practical Q Meter →



Application's →

(1) **Measurement of Q of the coil** → The test coil is connected between terminal's T_1 & T_2 & circuit is resonated by varying the capacitor C. The voltmeter across the capacitor is calibrated to measure Q of the test coil.

$$\text{True value of } Q (Q_T) = \omega L / R$$

$$\text{Measured of } Q = Q_m = \frac{\omega L}{(R + R_{sh})}$$

$$= \frac{\omega L}{R\,(1 + R_{sh}/R)}$$

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$$Q_m = \frac{Q_T}{(1 + R_{sh}/R)}$$

$$Q_T = Q_m \left[1 + \frac{R_{sh}}{R} \right]$$

R_{sh} is to be low, so that $Q_T = Q_m$
R_{sh} is in the range of $m\Omega$.

(ii) Measurement of unknown Capacitance →

The test capacitance is connected between T_3 & T_4 and the circuit is resonated to a frequency f_1

$C \to C_1$

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_T)}}$$

Remove C_T b/w T_3 & T_4 change the capacitor to the f2 value C_2 to get the same value of f_1. From the known values of C_1 & C_2 C_T is measured.

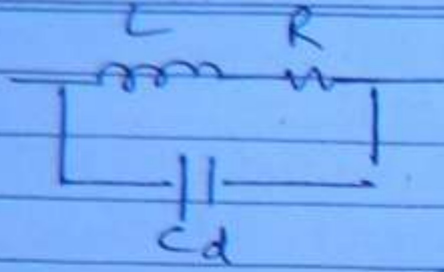
$C \to C_2$

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} \Rightarrow f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_T)}}$$

$$C_T + C_1 = C_2$$

$$\boxed{C_T = C_2 - C_1}$$

(iii) Measurement of distributed or self capacitance → (C_D).



$C \rightarrow C_2$

$f_2 = \frac{1}{2\pi\sqrt{L(C_2+C_d)}} = nf_1$

(154)

$C \rightarrow C_1$

$\frac{1}{2\pi} \frac{1}{\sqrt{L(C_2+C_d)}} = \frac{n}{2\pi\sqrt{L(C_1+C_d)}}$

$f_1 = \frac{1}{2\pi\sqrt{L(C+C_d)}}$

$C_d = \frac{C_1 - n^2C_2}{(n^2-1)}$

for $n=2$, $f_2 = 2f_1$

$$\boxed{C_d = \frac{C_1-4C_2}{3}}$$

Q In a Q meter an inductor is tunned to 2 MHz, 450 pf and 4 MHz with 90 pf. Than the distributed capacitance of the inductor is

a) 45
b) 90
c) 30 ✓
d) 360.

As

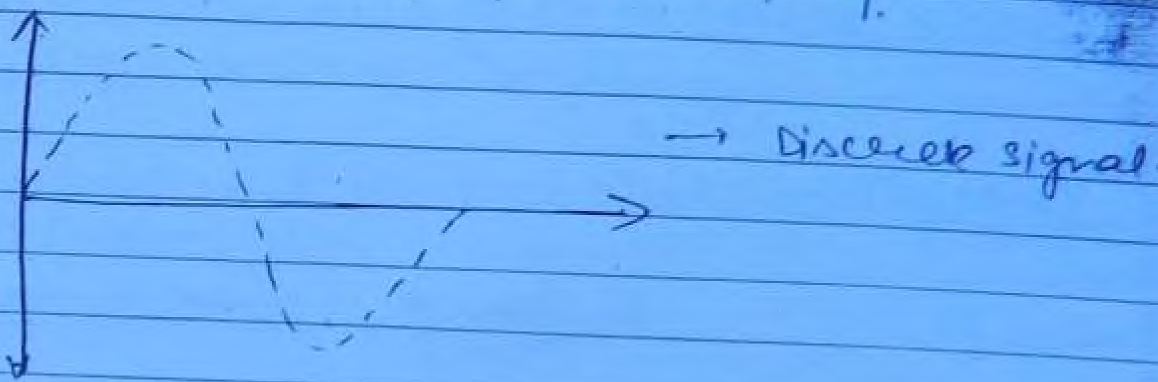
$f_2 = 2f_1$

$\Rightarrow 2(2)$

$\Rightarrow 4$

$C_d = \frac{450 - 4\times 90}{3} \Rightarrow 30\,pf$

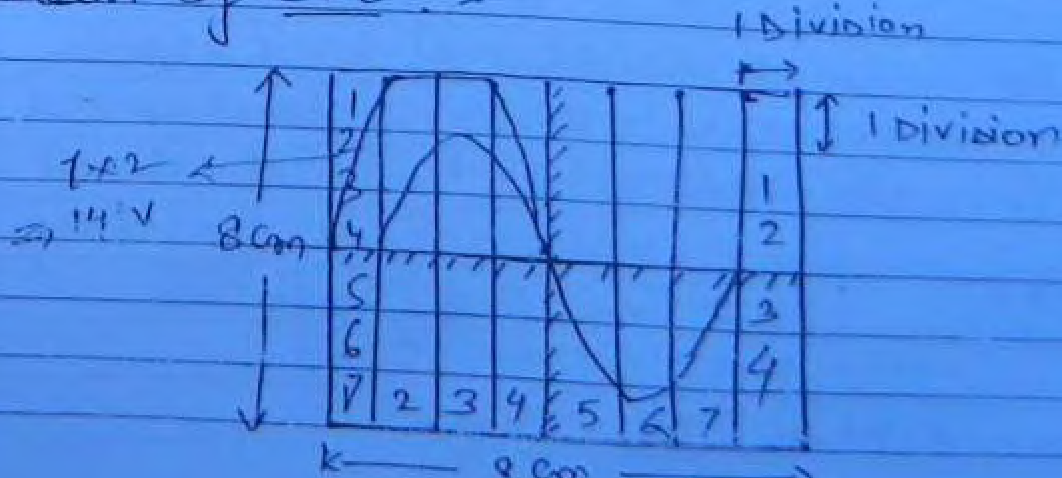
CRO → Cathode Ray Oscilloscope.



Advantages.

(i) X-Y plotter (wave form)
(ii) Vpp, Vm, VRMS
(iii) f, p, T
(iv) Compare the signal.
(v) Storage of data.
(vi) Scale is adjustable.
(vii) High Resolution
(viii) High Accuracy
(ix) High Sensitivity
(x) High speed
(xi) No external effect's.
(xii) Linear Device.

Screen of CRO :-



Measurement of Time period (T):-

$T = (X\text{-scale}) \times$ (No of Division's occupied by signal over one cycle).

X-scale or Line base or Time base → sec or msec/division or sec or msec/cm.

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Ex: X scale:- 2m sec/division

$X = 7 \times 2 = 14 \text{ m sec}$

$f = \frac{1}{T}$ Hz.

$$\text{No of cycles of waveform or signal visible on Screen} = \frac{\text{Total time period of screen}}{\text{Time period of waveform}}$$

Measurement of Vpp:-

$V_{PP} = (Y\text{-scale}) \times$ [No of division occupied by signal from +ve peak to -ve peak].

Yscale → volt division or ~~cm/division~~. Volts/cm

Ex:- Yscale:- 2V/cm

$V_{PP} = 4 \times 2 = 8V$

peak:- $V_m = \frac{V_{PP}}{2} = 8/2 \Rightarrow 4V$

Sinusoidal:- $V_{RMS} = \frac{V_m}{}$

If I/P signal applied having peak to peak more than max^m allowable peak to peak signal than signal will be clipped on the screen.

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Q14

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Xscale = 0.5 ms/cm → Time period of screen ⇒ 10 × .5 ⇒ 5 msec

Yscale = 100 mV/cm.

time period of signal = $\frac{1}{200}$ Hz = 5 msec.

$n = \frac{5}{5} \Rightarrow$ 1 cycle.

→ 8 × 100 ⇒ 800 mV (max on screen)

Signal :- $V_{RMS} = 300\ mV$

$V_m = \sqrt{2}\ V_{RMS}$ ⇒ 2

$V_{pp} = 2\sqrt{2} \times 300$

⇒ 848 mV > 800 mV

Calibration of CRO →

CRO is calibrated by applying a square or pulse signal having a frequency of 1 KHz and an amplitude of 1 volt.

Bandwidth of CRO → The max^m frequency of the signal that can be visible on the screen without any distortion is called bandwidth. Bandwidth and rise-time of the signal are related as

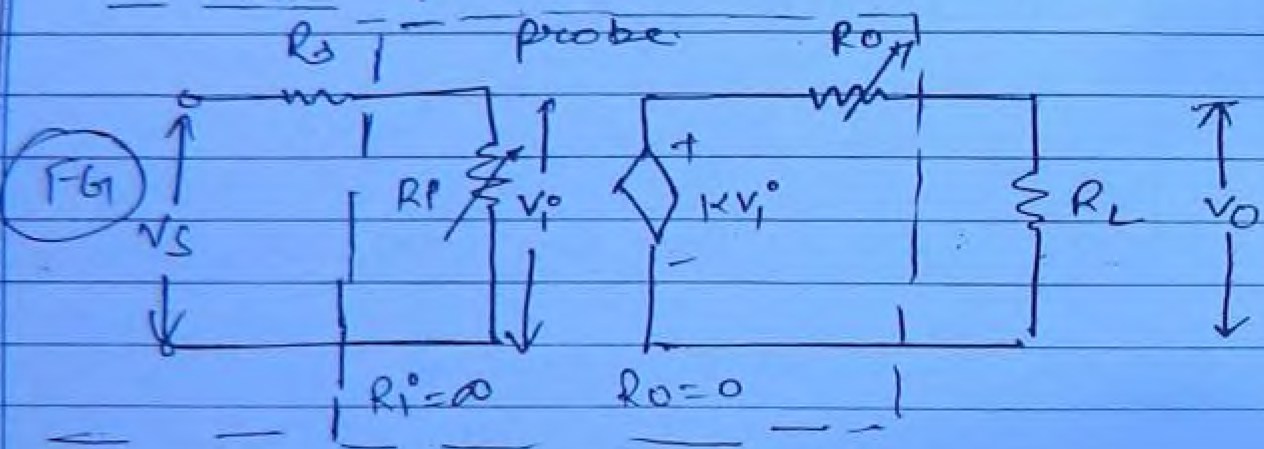
$$\boxed{B_W \times t_r = 0.35}$$

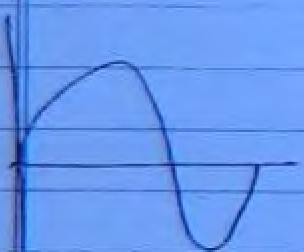
B_W = Bandwidth in Hz

t_r = Rise time in sec.

Input Impedance of CRO →

The input resistance of CRO should be very high of order of 1 MΩ and the input capacitance should be low of the order of pF so that the input voltage signal will not be attenuated.





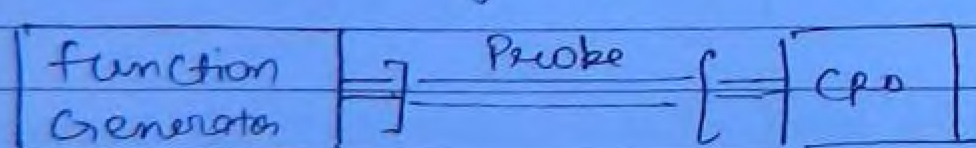
$\boxed{V_i = V_s}$ $\boxed{V_o = KV_i}$

$\boxed{V_o = KV_s}$

If $K = 1$

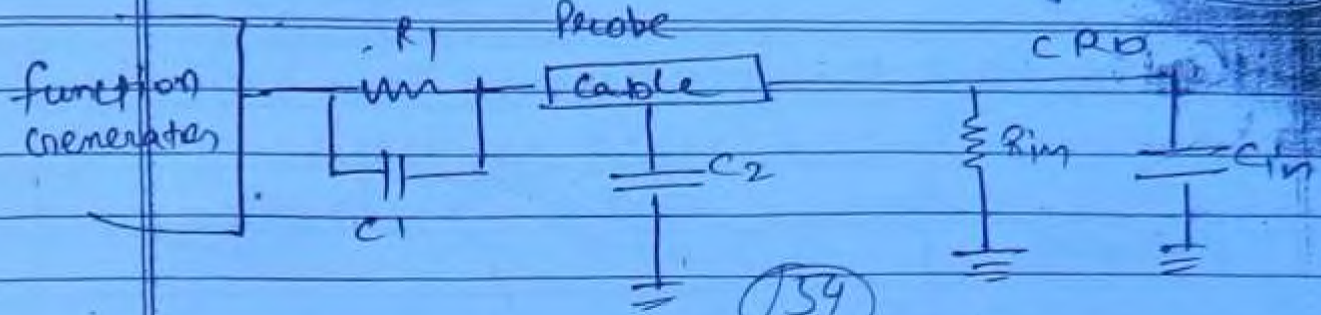
$\boxed{V_o = V_s}$

Transfer + Resistor = Transistor.



10X a high Impedance probe

Electrical Equivalent ckt.



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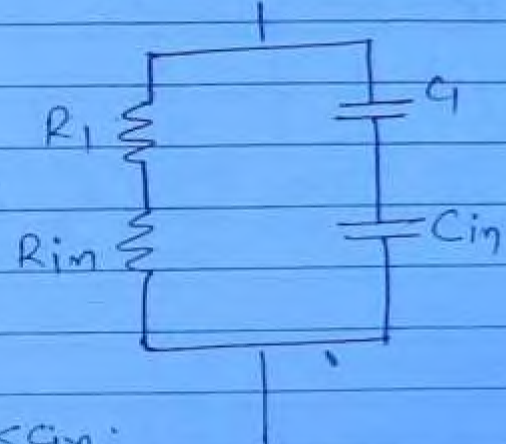
$\dfrac{R_1 + R_{in}}{R_{in}} = 10$

$\boxed{R_1 = 9 R_{in}}$

Independent of frequency

$\boxed{R_1 C_1 = R_{in} C_{in}}$



$R_1 C_1 \rightarrow$ Resistance & Capacitance of probe

$R_{in}, C_{in} \rightarrow$ Resistance & capacitance of CRO.

$C_2 \rightarrow$ capacitance to ground $<< C_{in}$.

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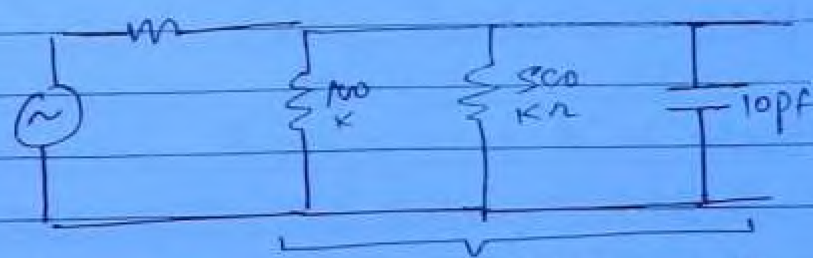
Q3



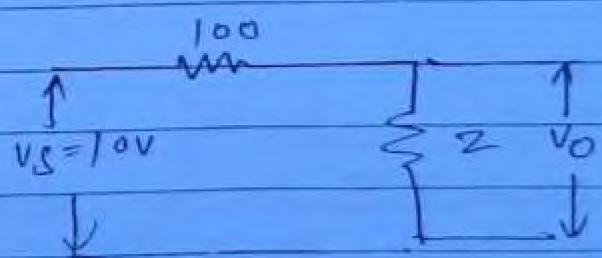
$X_C = \dfrac{1}{2\pi f C}$

$\Rightarrow \dfrac{1}{2\pi \times 100 \times 10^{3} \times 10}$

$Z = 100 \parallel 500 \parallel (-j159\,k)$

$X_C = j159\,\Omega$

$Z \Rightarrow \dfrac{500 \times 100}{600} \parallel j159$.



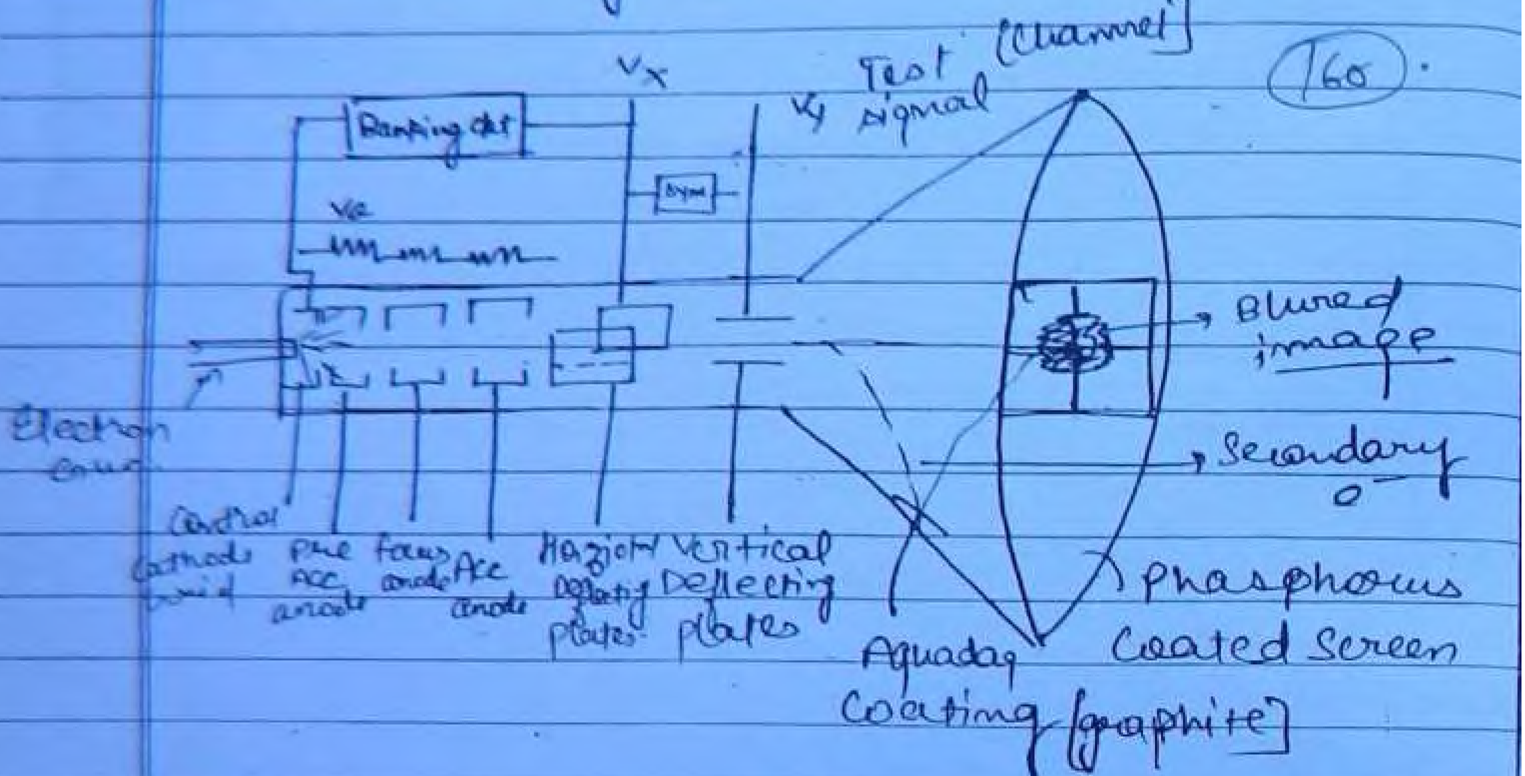
$\Rightarrow \dfrac{V_S \cdot Z}{100 + Z}$.

$\dfrac{\cancel{500 \times 100 \times j159}}{\dfrac{600}{500 \times 100} + \dfrac{1}{j159}}$

$600(j159) + 500 \times 100$

$(500 \times 100)(j159)$

Cathode Ray Tube:-



CRO or CRT working on the principle of Thermionic emission i.e emission of electron's from a heated surface. (electron gun)

Control cathode grid is applied with negative potential which is used to control no of electron's entering into other region of CRT.

$$KE = PE$$

$$\frac{1}{2} m v^2 = q V_a$$

$$v = \sqrt{\frac{2 q V_a}{m}}$$

$$v \propto \sqrt{V_a}$$

v = velocity of electron
V_a = anode voltage.

The pre accelerating and accelerating potential are used for controlling the speed of the electron shown above.

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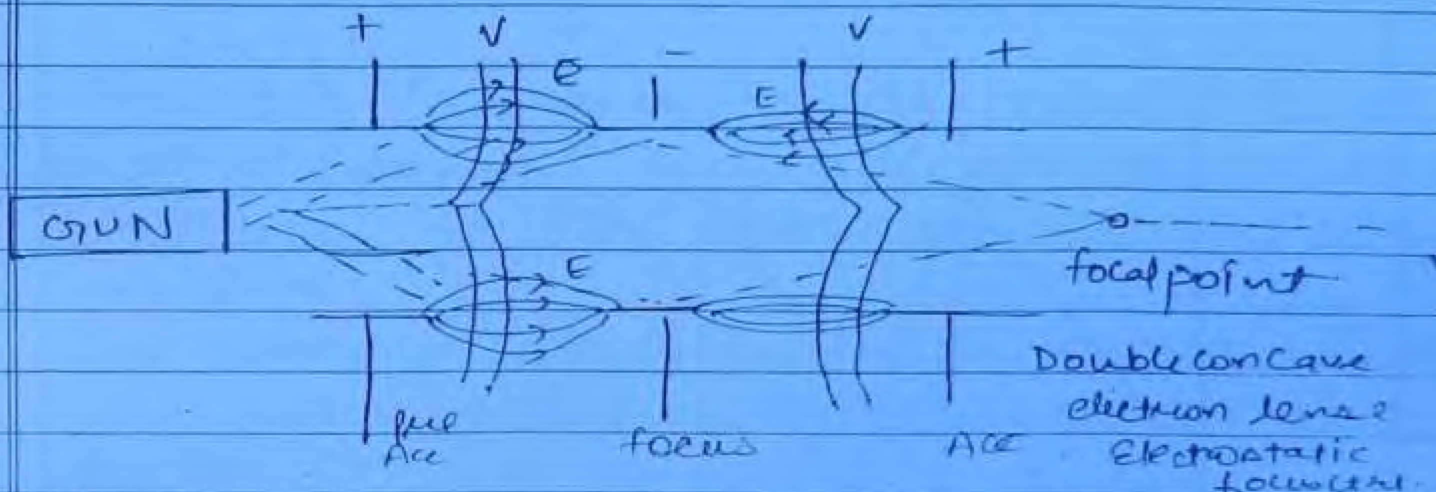
In case of High frequency CRO post accelerating anode is used for controlling the brightness of electron beam on the screen for the frequency above 10MHz.



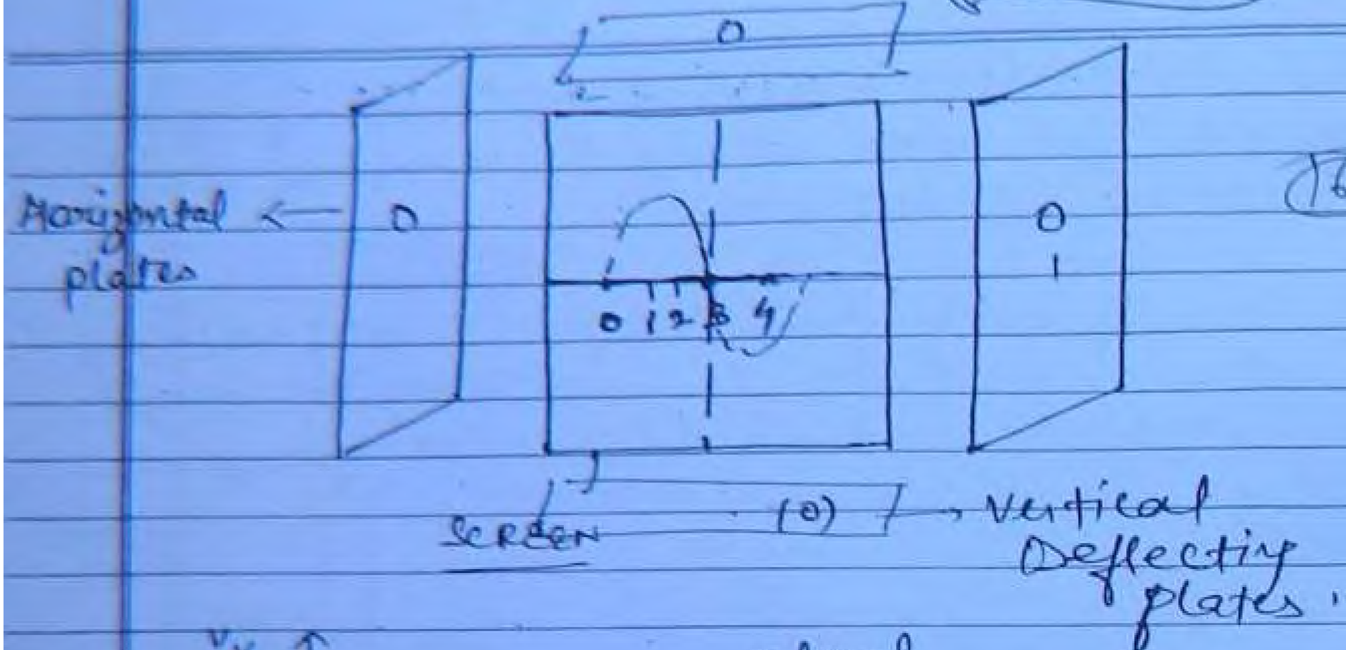
Electrostatic focus control is used in laboratory CRO's.

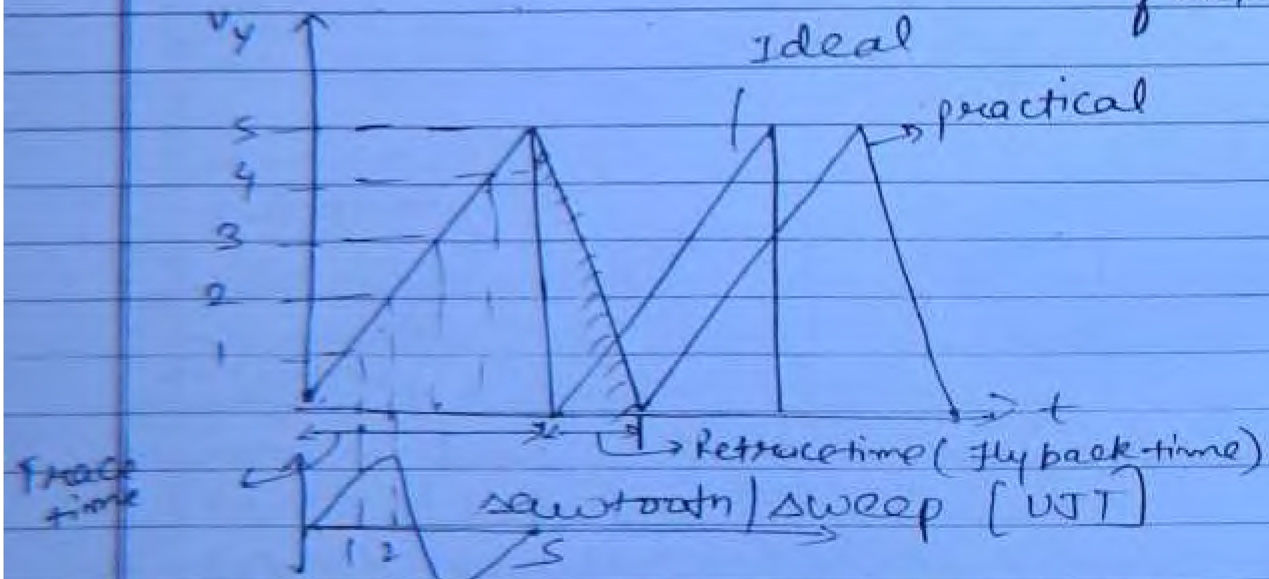
Electromagnetic focus control is used in television picture tube

Both electrostatic & magnetic focusing's are used in case of computer CRO's.

Electrostatic focusing working on a principle of double concave electron lense adjustment for controlling of focal point.

Astigmatism is a method used for fine focus control by applying a small dc potential and by removing voltage sources of horizontal and vertical deflecting plate potential's.





The horizontal Deflecting plates are in vertical position and is applied with sawtooth or sweep voltage which is produced by the UJT relaxation oscillator.
During Retrace period of this waveform blanking ckt is initiated which gives a comment to the cathode grid to produce high negative potential so that electrons are blanked out entering inside of CRT.
The time period of sawtooth waveform is adjusted by using synchronizing ckt

so that the time period of external signal and horizontal signal are to be synchronizing.

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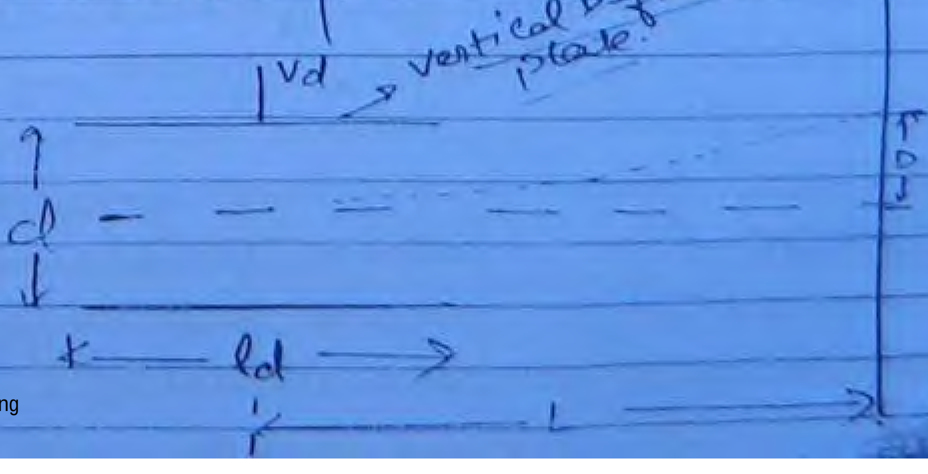
vertical deflecting plates are kept in horizontal position. A test signal is applied to the vertical deflecting plates so that the electron beam path is in synchronism with the test signal and this is achieved by controlling synchronizing circuit.

Aquadag Coating → Aquadag coated with graphite is used for collection of secondary electron's so that keep the CRT electrically neutral.

The screen is coated with phosphorus depending on phosphours coating the colour of the beam on the CRO is changed.

- P_{11} → General purpose CRO
- P_4 → Black & white CRO
- P_6 → Colour display
- P_{15} → sampling CRO (High freq CRO)
- P_{31} → storage CRO.

Deflection Sensitivity →



Date
Page

$$D = \frac{L\, l_d\, V_d}{2 d V_a}$$

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Deflection sensitivity $S = \frac{D}{V_d} = \frac{L\, l_d}{2 d V_a}$ mm/v

Deflection factor $G = 1/S$.

D = Deflection height on the screen.
d = distance b/w plates.
ld = Length of plates.
L = Distance b/w center of plate to screen.
Va = Anode voltage.
Vd - Deflecting plate voltage.

Lissajous pattern → If both horizontal & vertical deflecting plates are applied with sinusoidal signal's the waveform pattern appearing on the screen is called lissajous pattern.

Application's →

(i) It is used for finding
(i) phase angle difference b/w horizontal, vertical voltages applied.
(ii) The frequency ratio b/w the horizontal & vertical ~~from a known frequency~~
From the known frequency of one of the signal other signal frequency can be calculated by using Lissajous pattern.

Note:- At any point of time the electron beam on the screen is the vector sum of the voltage applied to the horizontal & vertical deflecting plates.
The vector sum consisting of both magnitude and phase angle.

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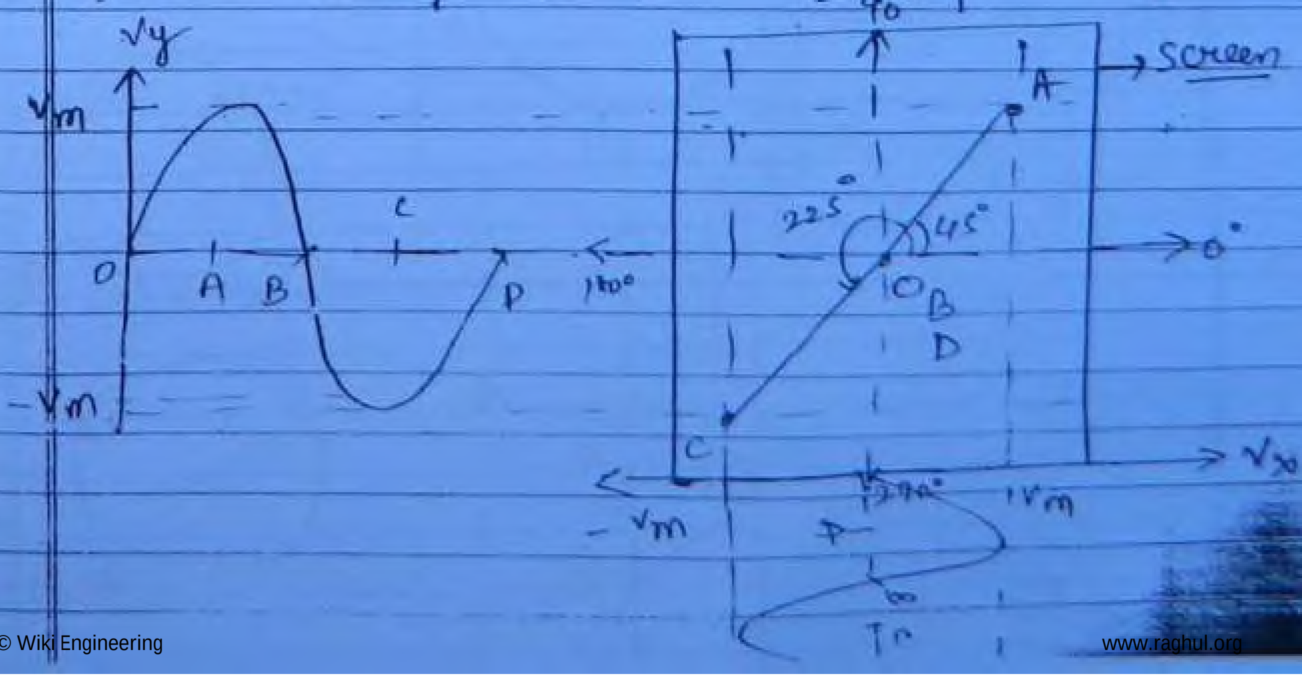
$$V_x = V_m \sin \omega_x t$$
$$V_y = V_m \sin(\omega_y t + \phi)$$

Case I:- Both V_x & V_y have same frequency ($\omega_x = \omega_y = \omega$) but phase difference ($\phi$):- [$V_x = V_m \sin \omega t$, $V_y = V_m \sin(\omega t + \phi)$]

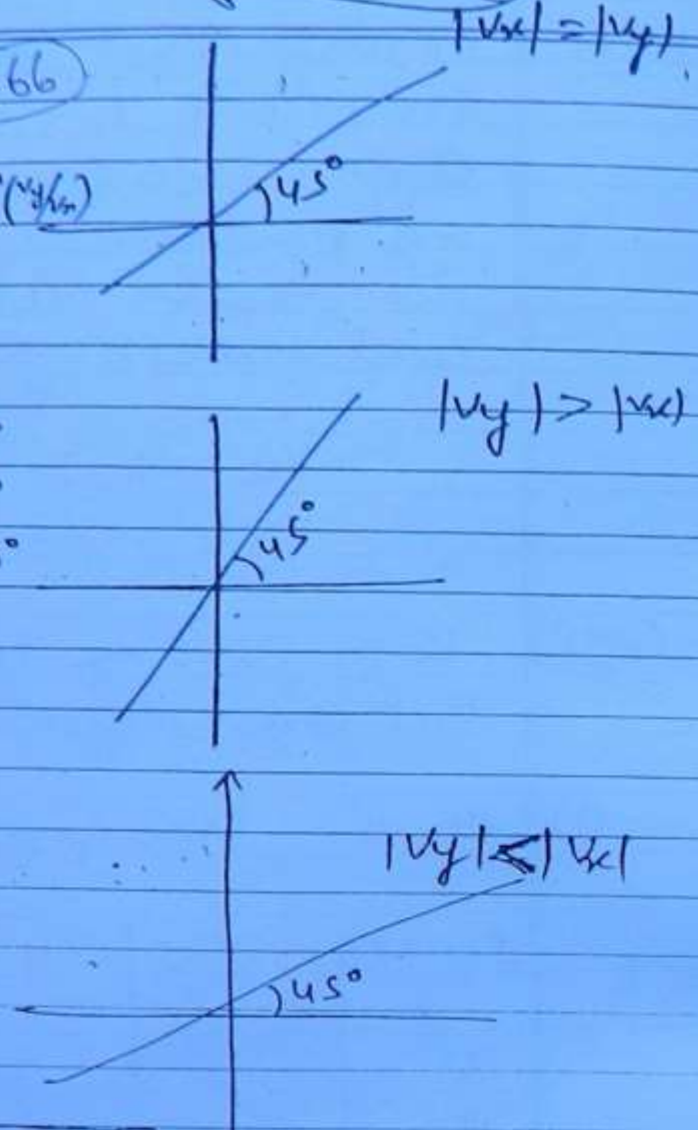
a) $\phi = 0°$ ($V_x = V_m \sin \omega t$, $V_y = V_m \sin \omega t$).

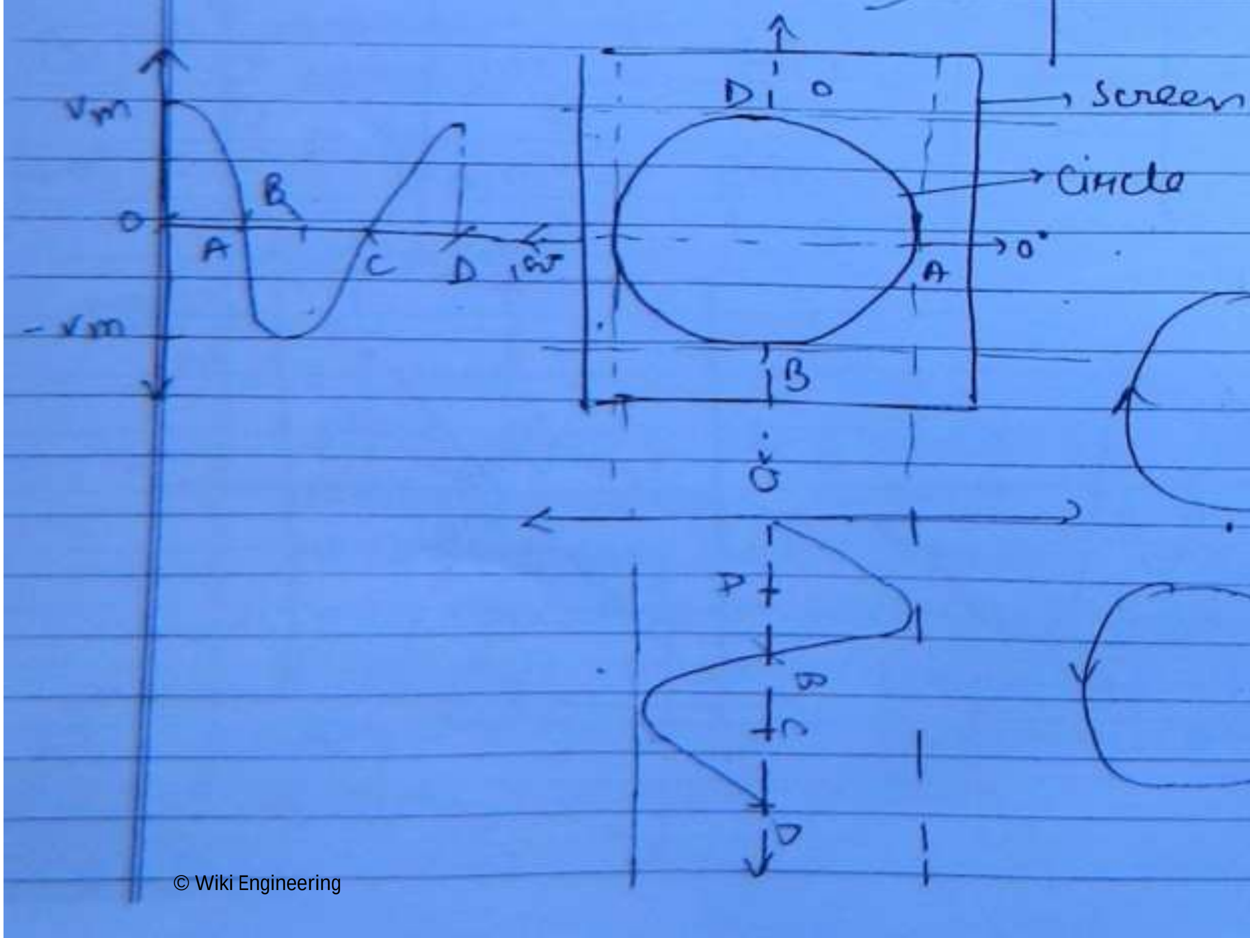
Point	V_x	V_y	$\sqrt{V_x^2 + V_y^2}$	$\theta = \tan^{-1}(V_y/V_x)$
O	0	0	0	0
A	V_m	V_m [2V_m]	$\sqrt{2}\, V_m$ [$\sqrt{5}\, V_m$]	45° [63°]
B	0	0	0	0
C	$-V_m$	$-V_m$ [$-2V_m$]	$\sqrt{2}\, V_m$ [$\sqrt{5}\, V_m$]	225° [180+63]
D	0	0	0	0

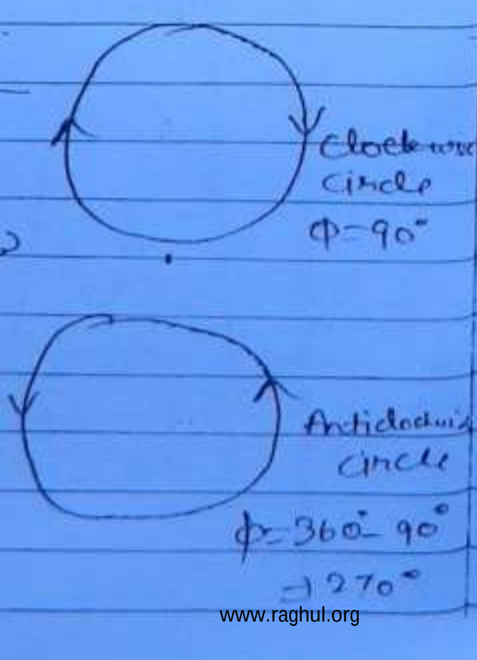


b) $\phi = 90^\circ$. (166)

Point	V_x	V_y	$\sqrt{V_x^2+V_y^2}$	$\theta = \tan^{-1}(V_y/V_x)$
O	0	V_m	V_m	90°
A	V_m	0	V_m	0°
B	0	$-V_m$	V_m	270°
C	$-V_m$	0	V_m	180°
D	0	V_m	V_m	90°







LP for different ϕ →

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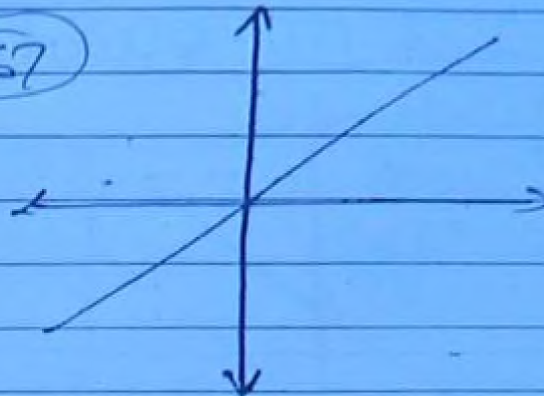
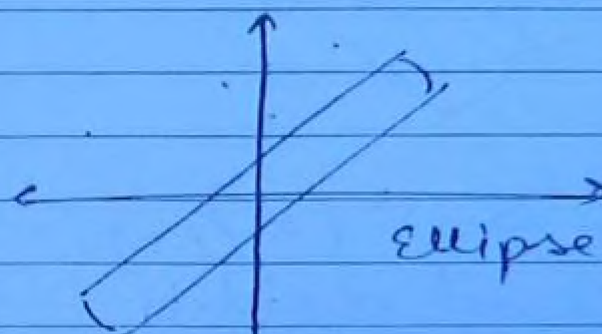
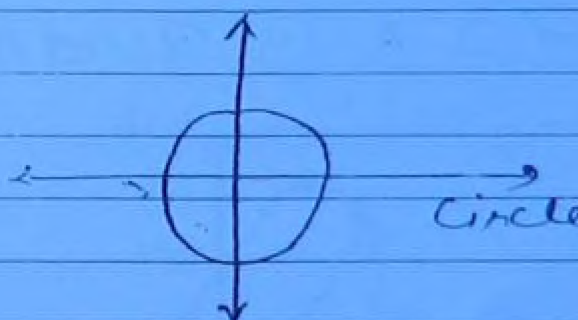
$\phi = 0°$ or $360°$



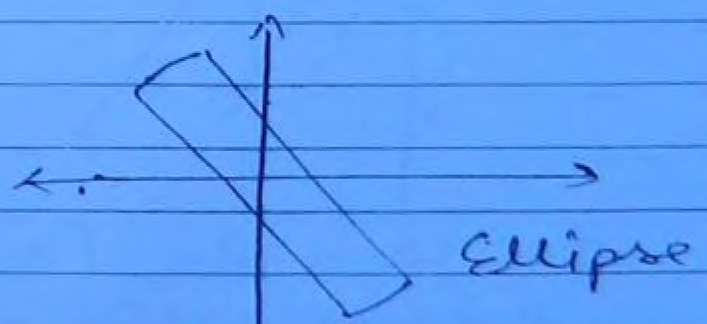
$0 < \phi < 90°$
or
$270° < \phi < 360°$



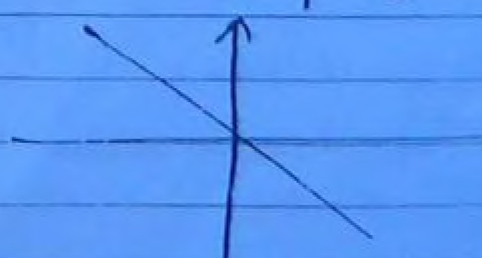
$\phi = 90°$ or
$270°$



$90° < \phi < 180°$
or
$180° < \phi < 270°$



$\phi = 180°$



Calculation of ϕ from LP →

(1) LP in I & III Quadrent's →

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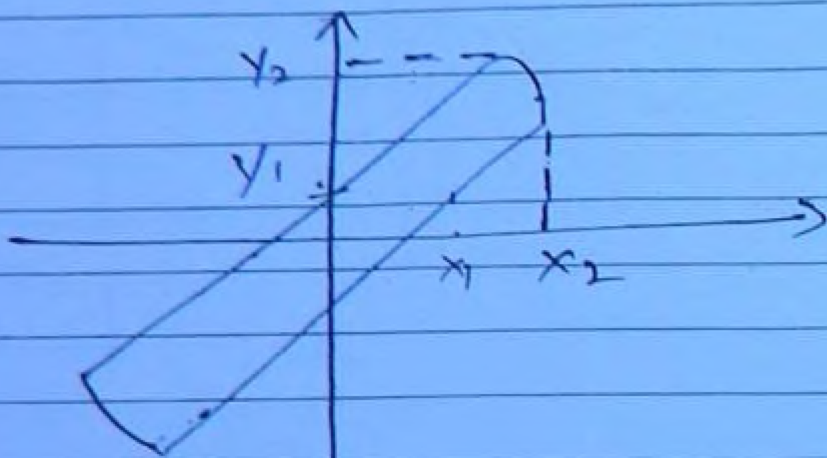
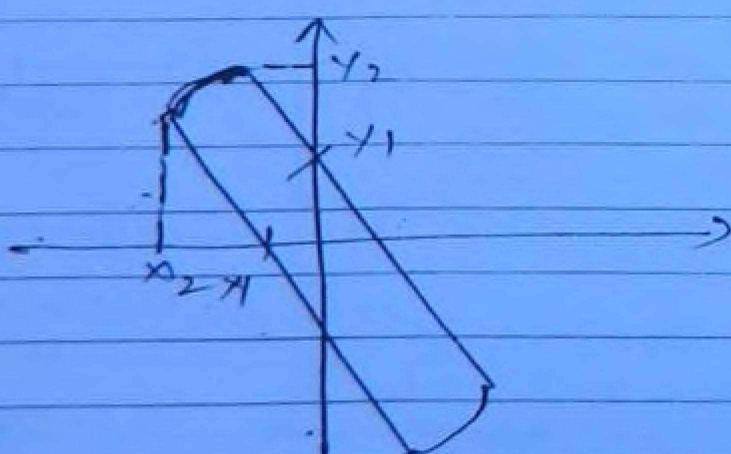


$\phi = \sin^{-1}(x_1/x_2)$ Or $\sin^{-1}(Y_1/Y_2)$

Second possibility $= \cancel{(180-\phi)}$ $(360-\phi)$

(2) LP in II & IV Quadrent's →



$\phi = 180 - \sin^{-1}(x_1/x_2)$ (Or)

$\phi = 180 - \sin^{-1}(Y_1/Y_2)$

Second posiblity $= \cancel{(180-\phi)}$ $(360-\phi)$

Q For the following Lissagous pattern find ϕ

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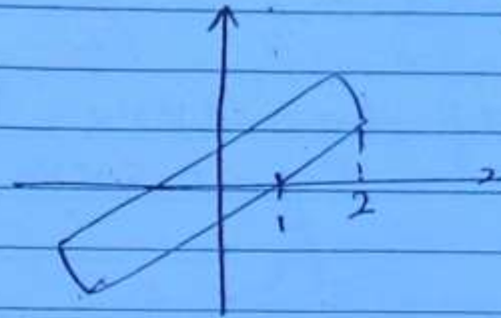
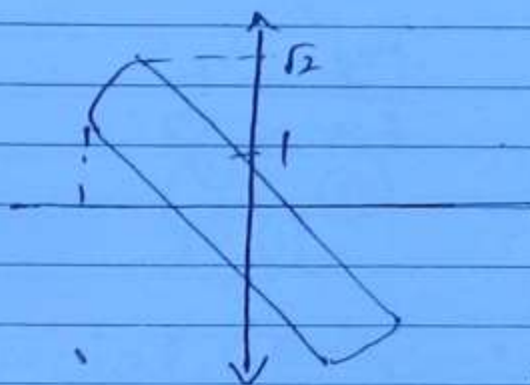
(i)



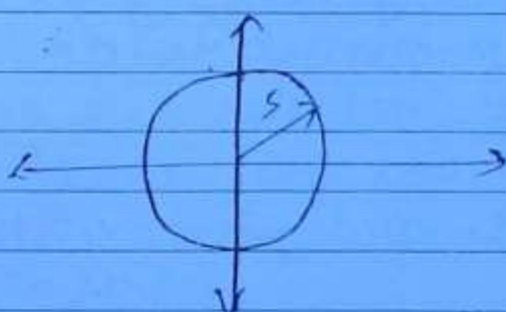
$\phi = \sin^{-1}(1/2) = 30^\circ$
or
$360^\circ - 30^\circ = 330^\circ$.

(ii)



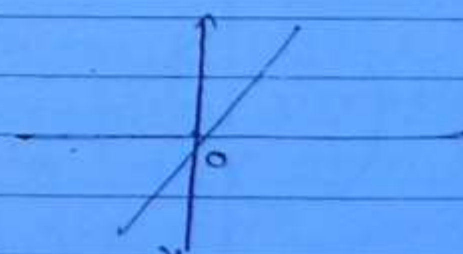
$\phi = 180^\circ - \sin^{-1}(1/\sqrt{2})$
$\Rightarrow 135^\circ$ or
$\Rightarrow 360 - 135$
$\Rightarrow 225^\circ$

(iii)



$\phi = \sin^{-1}(5/5) = 90^\circ$ or 270°

(IV) $\phi = \sin^{-1}(0/0) = 0^\circ$
or
360°

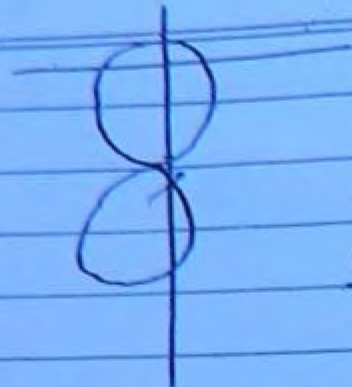


Case 2nd

V_x, V_y has different frequency.

$V_x = V_m \sin \omega_x t = V_m \sin(2\pi f_x) t$
$V_y = V_m \sin \omega_y t = V_m \sin(2\pi f_y) t$

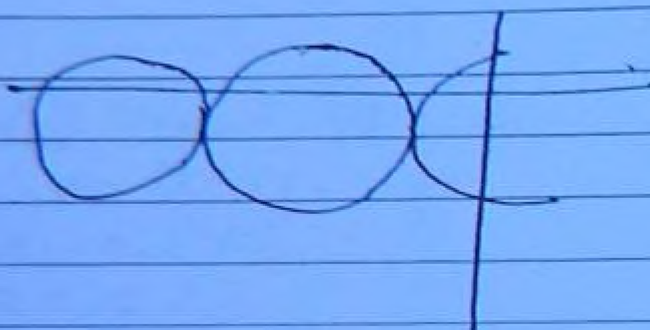
$$\frac{\omega_y}{\omega_x} = \frac{f_y}{f_x} = \frac{\text{No of horizontal Tangencies}}{\text{No of vertical Tangencies}}$$



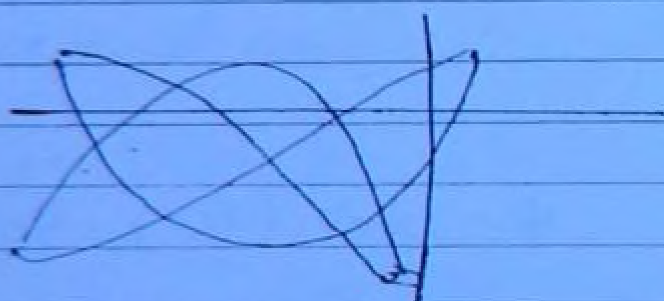
$$\frac{f_y}{f_x} = \frac{2}{4} \Rightarrow \frac{1}{2}$$

If $f_x = 1\,KHz$

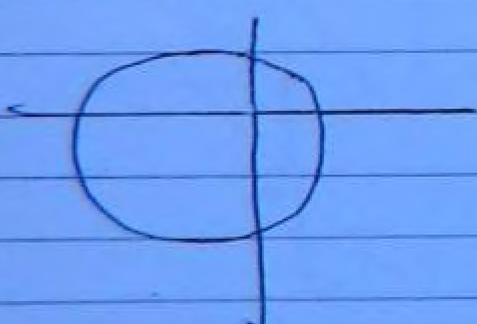
$f_y = 0.5\,KHz$.



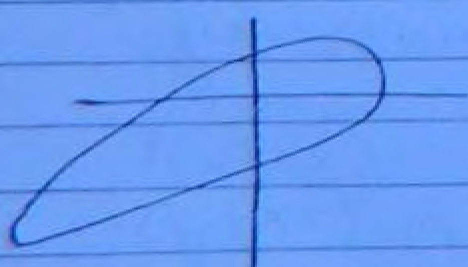
$$\frac{f_y}{f_x} = \frac{5}{2}.$$



$$\frac{f_y}{f_x} = \frac{6}{4} \Rightarrow \frac{3}{2}.$$



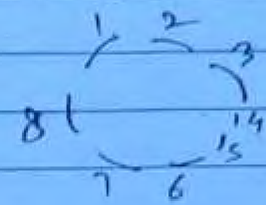
$$\frac{f_y}{f_x} = \frac{2}{2} \Rightarrow 1$$



$$\frac{f_y}{f_x} = \frac{2}{2} \Rightarrow 1$$

$$\frac{f_y}{f_x} \Rightarrow \frac{1}{1} \Rightarrow 1.$$

Special cases →



$\frac{f_y}{f_x} \Rightarrow 1/8$

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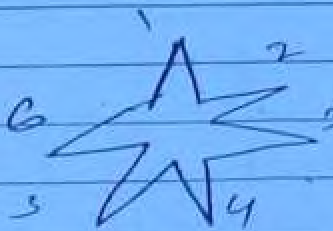
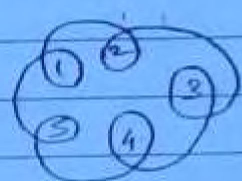


$\frac{f_y}{f_x} \Rightarrow 1/6$

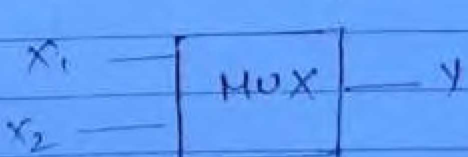


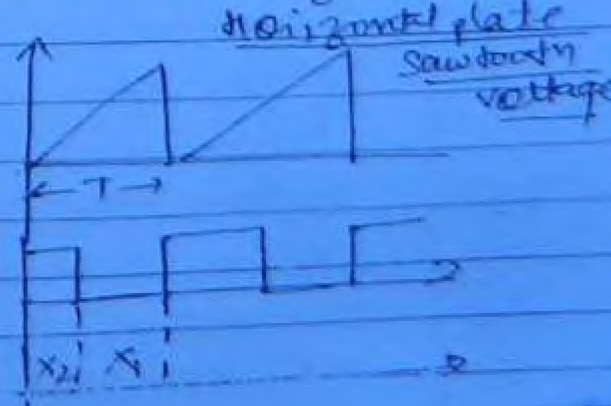
$\frac{f_y}{f_x} = 1/5$

Special CRO's :-

1. Dual trace CRO :- It contain's two vertical deflecting plates wrt and one e⁻ gun is the selection switch is used for the selection of one of the input signal so that alternatively signal's can be absorbed on the CRO screen.

A Dual trace CRO with multiplexer is used to display 2 input waveform simaltenously on the screen by Repping the time period of selection switch half the time period of saw tooth signal applied to horizontal plate





S	Y
0	X_1
1	X_2

Date
Page

(2) Dual Beam CRO :— It contain's two vertical deflecting plates and two electron gun's So that the two signal applied to the vertical plates can be simaltenously visible on the screen.
Cost of the dual beam is higher.

(3) High frequency CRO :— post acclerating anode is provided for ↑sing the brightness of electron beam.
→ It has low input capacitance
→ short persistant (P_{15}) phosphorus is used on the screen.

(4) Storage CRO :— Memory storing element are provided for storage of signal's
→ If the more storage data is required than it is converted into digital and stored in digital format.

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01/2012

Digital Meters :— [Digital voltmeter]

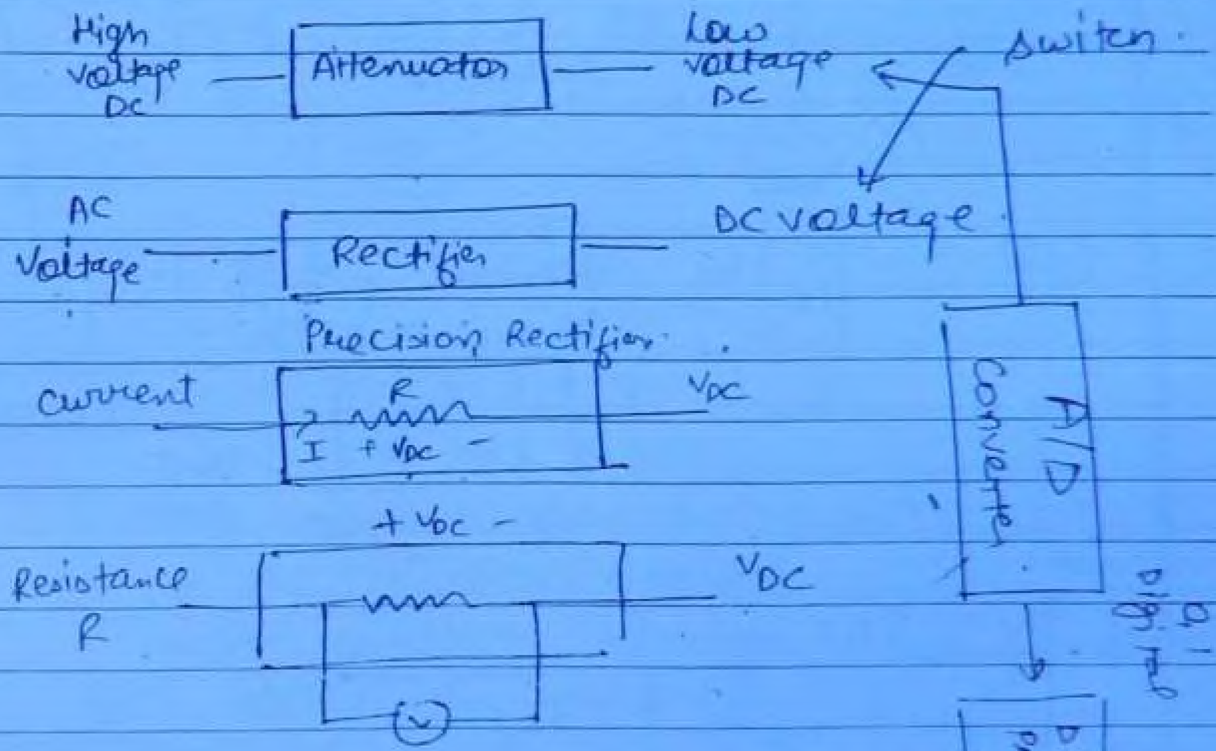
Advantages :—

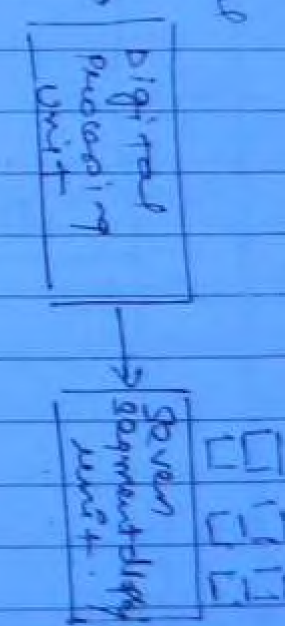
1. No parallel error.
2. Multiple application.
3. High speed
4. Store the data
5. Interfacing is possible
6. High Resolution.
7. High sensistivity
8. High accuracy
9. power consumption is low
10. Compact in size
11. No external effect's.

(12) Maintenance Free.

Disadvantage

(i) External battery source is required.





A/D Converter	Conversion Time [cycles]
(1) flash	1
(2) SAR	n clocks
(3) Ramp or Counter	2^n clocks
(4) Dual Scope	$2^{(n+1)}$ clocks.

where, n = No of digital output bits.

The basic measurable quantity of an A·D converter is dc voltage.

Dual slope A/D Converter :-

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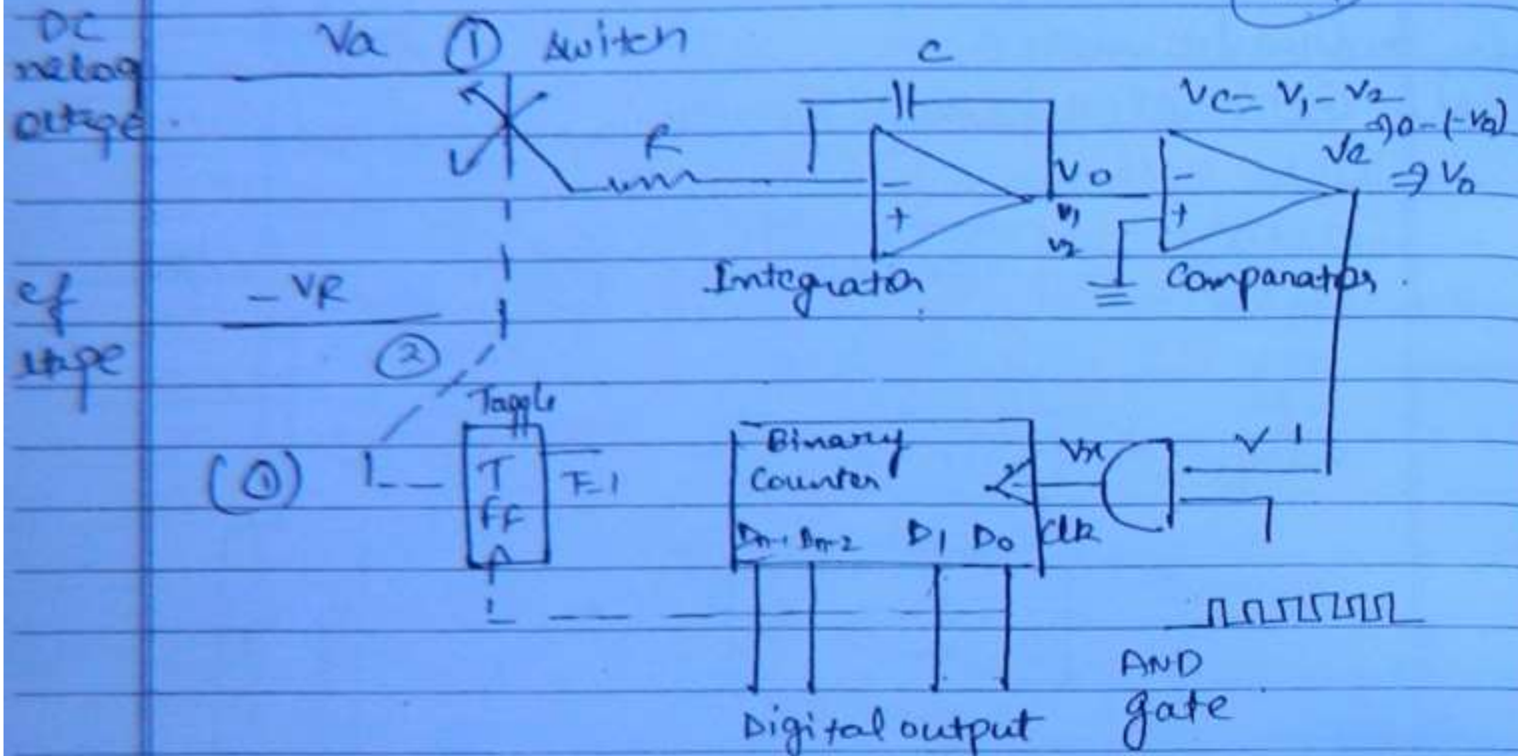

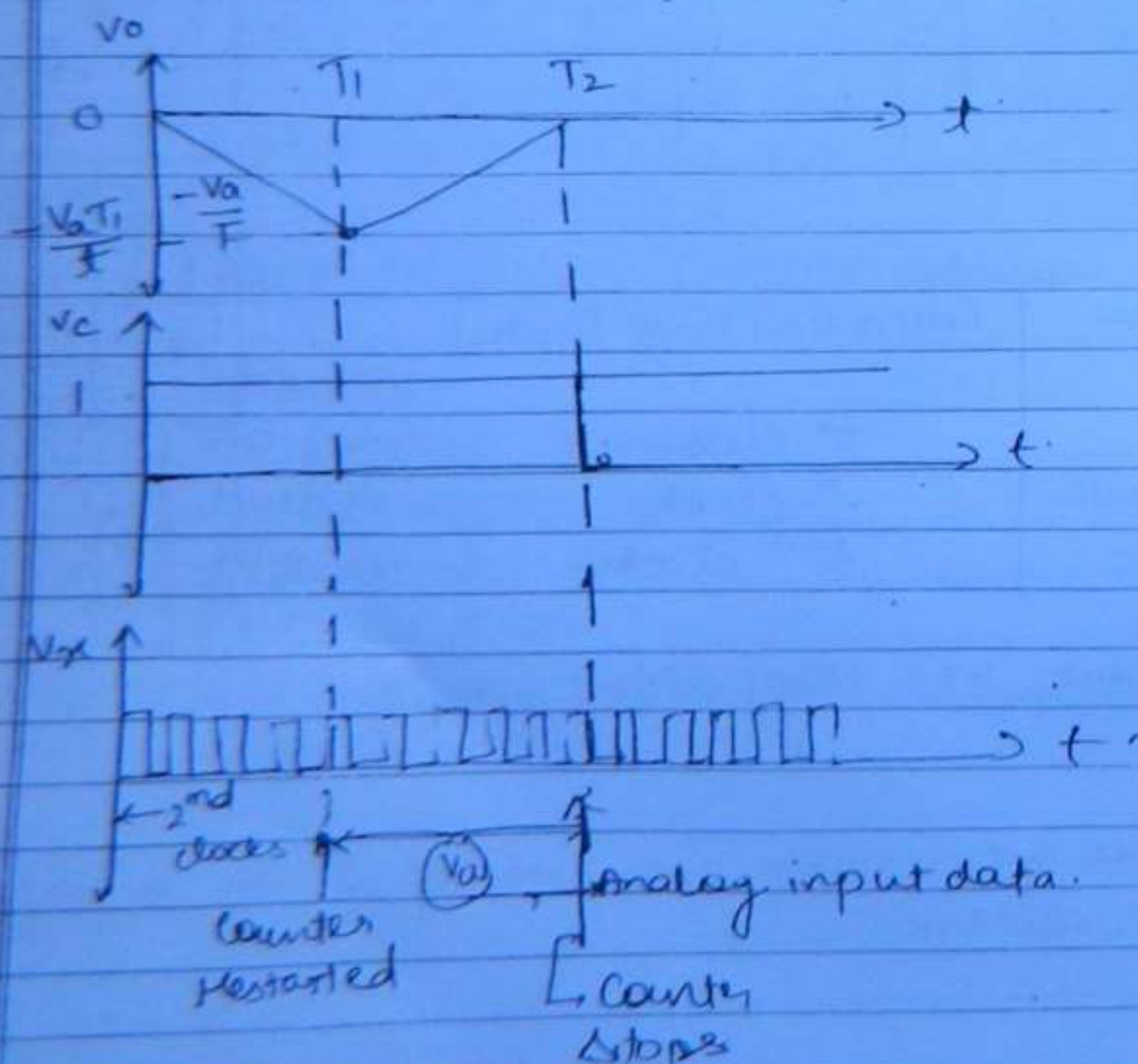

Switch is at posn assume all output's of the device's integrater and gate binary counter T flip flop are initially zero.

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at time $t=0$; switch is at (1)

$$V_0 = -\frac{1}{RC}\int V_a \cdot dt$$

$$\Rightarrow -\frac{V_a \cdot t}{\tau} \qquad \tau = RC$$

$$V_0 = m t, \qquad m = -\frac{V_0}{\tau}$$

$V_0 = -ve \Rightarrow V_c = +ve = 1$

$V_c = 1 \Rightarrow V_x = $ clocks.

Binary counter incremented upto $t = T_1$.

At, $t = T_1 = (2)^n$ clocks applied.

Counter reset's to '0' value and T FF, output $Q=1$, switch changes to (2).

At $t = T_1$, $\boxed{V_0 = -\frac{V_a T_1}{\tau}}$

$0 < t < T_1 \rightarrow$ first Integration or Integration.

At $t = T_1$, switch at (2)

$T_1 < t < T_2 \rightarrow$ Second integration

$$V_0 = -\frac{V_a T_1}{\tau} + \left[-\frac{1}{\tau}\int_{T_1}^{t}(-V_R)\,dt\right]$$ or Deintegration

$$\boxed{V_0 = -\frac{V_a T_1}{\tau} + \frac{V_R}{\tau}(t - T_1)}$$

$$\boxed{\begin{array}{l}\text{at } t = T_2,\ V_0 = 0. \\ V_0 = -\frac{V_a T_1}{\tau} + \frac{V_R}{\tau}(T_2 - T_1)\end{array}}$$

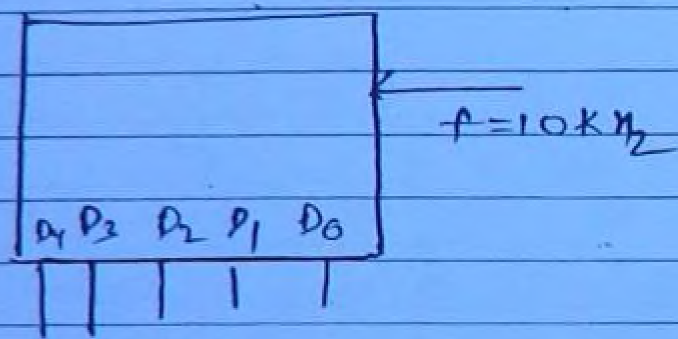
$$\boxed{V_R(T_2 - T_1) = V_a T_1}$$

$$(T_2 - T_1) = \frac{V_a T_1}{V_R} = \frac{V_a \cdot 2^n}{(2)^n} = V_a \cdot$$

$$\therefore \boxed{V_R = 2^n}$$

T76

Max^m Convertion Time $\rightarrow$ $2^n + 2^n = (2)^{n+1}$ clock's



$n = 5$

$T_1 = 2^n \times T_{clk}$

$\Rightarrow (2)^5$

$\Rightarrow 10 \times 10^3$

$\boxed{T_1 = 3.2 \, msec}$

$T_1 = 10/50 \, sec$

$V_R = 2V$

$V_a = 1V$

$T_2 = ?$

$V_a T_1 = V_R (T_2 - T_1)$

$(1) \frac{10}{50} = 2 (T_2 - 10/50)$

$\frac{10}{50} = 2 (\cancel{50} T_2 - 10/50)$

$\frac{1}{10} = T_2 - 10/50$

$T_2 = 1/10 + 10/50 \Rightarrow \frac{5+10}{50} \Rightarrow \frac{15}{50} = \frac{3}{10} = 0.3$

Q20.

$V_R = 100\ mV$

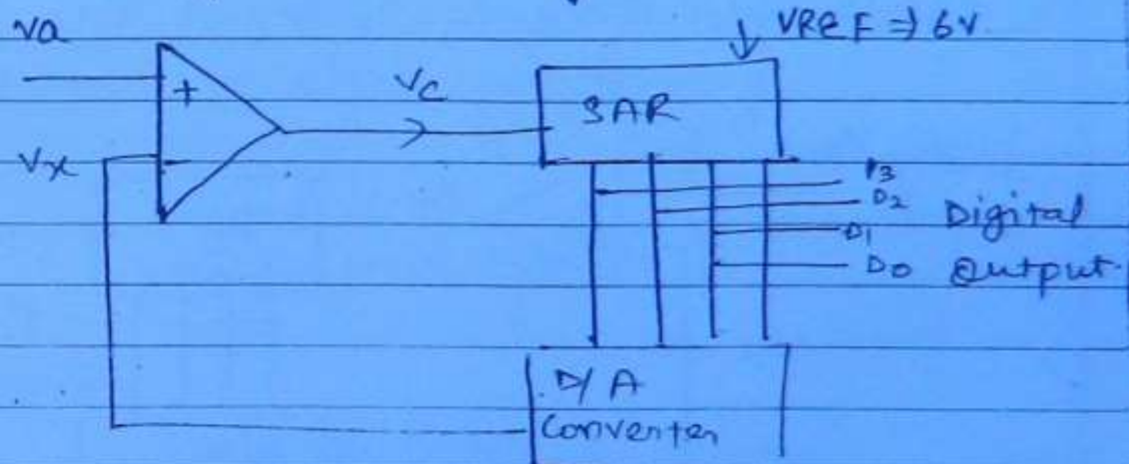
$T_1 = 300\ msec$

$T_2 - T_1 = 370.2\ msec$ (177)

$Va = ?$

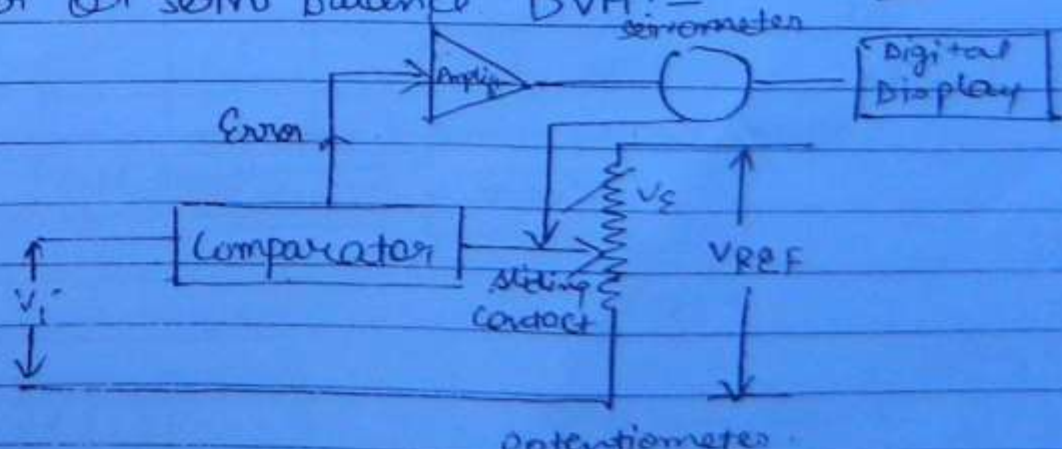
$$Va = \frac{V_R(T_2 - T_1)}{T_1} \rightarrow 123.4\ mV.$$

(Successive Approximation Register) SAR :-

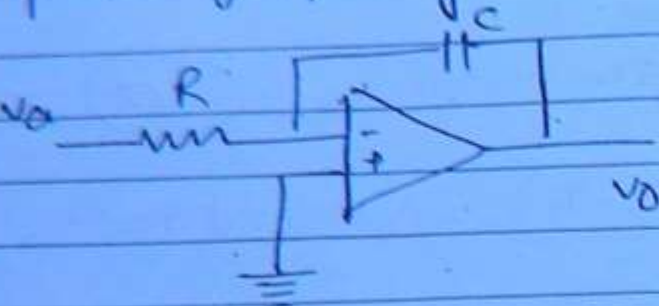


$(2)^3$	$(2)^2$	$(2)^1$	$(2)^0$		
D_3	D_2	D_1	D_0		
1	0	0	0		
6	3	1.5	.75	⇒ 11.25	Va
1	0	1	0	→ 7.5	V

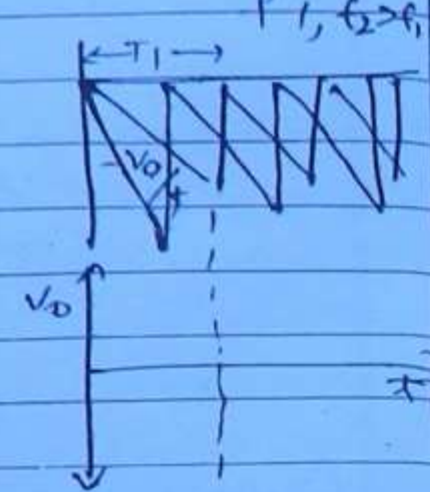
Counter Or Servo balance DVM :- (Comparator or Self balance type DVM :-)



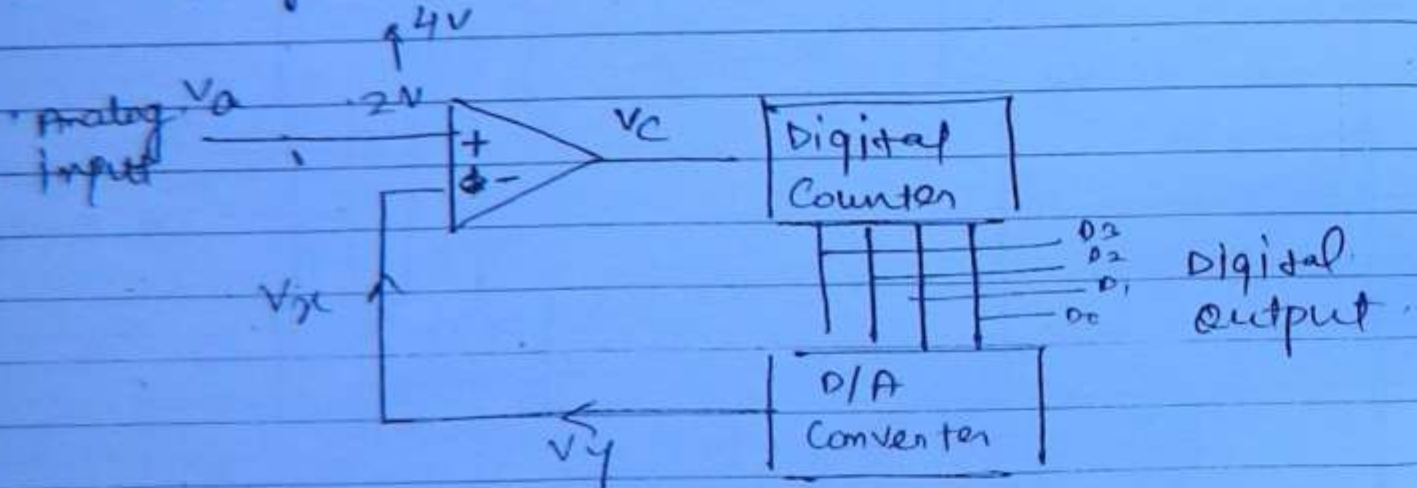
Voltage To frequency Counter →

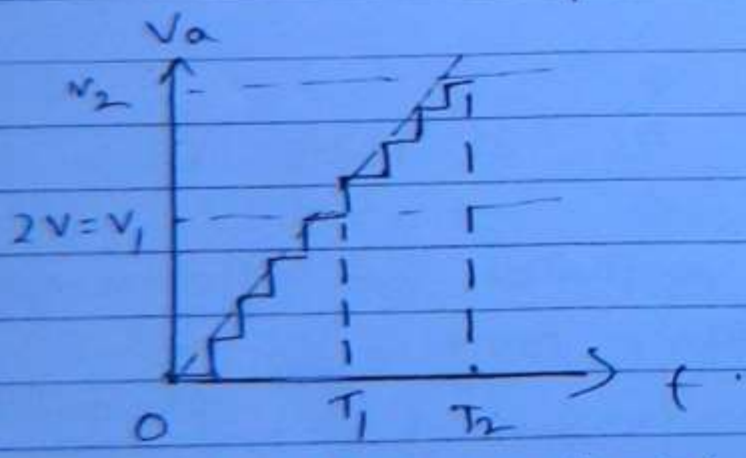


$V_0 = -\frac{Va\,t}{T}$



Ramp Type DVM :—





Voltage to time converter.

$\boxed{\cancel{V \propto T}}$ $\boxed{T \propto V}$

Parameter of Digital Meter →

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(i) <u>Resolution</u> → Smallest value of I/P that can be measured by digital meter is called resolution.

$$R = \frac{1}{(10)^n}$$

n = No of full digit's.

(ii) Sensitivity (S) → Smallest value of input that can be able to measure by digital meter at a given range is called sensitivity.

S = Resolution × min^m full scale value

$S = R \times (FS)_{min}$

$$\boxed{S = R \times \text{Range of meter}}$$

(iii) Total <u>Error</u> →

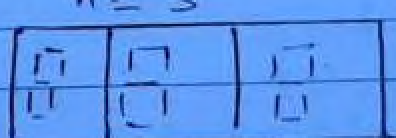
⇒ [x % of Reading value] + Error at counter value

x % → Error.

(iv) Over <u>Ranging</u> → The extra ½ digit is switched ON is called over ranging. ½ digit

$n = 3$

(Resolution)



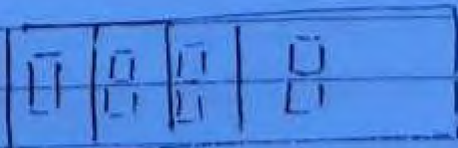
Full Digit's

$\frac{1}{10^3} = .001$

000 — 999

0-1 V Range

000 - 999



$\frac{1}{10^3}$ .001

0-1V Range

0000 - 1999

0000 - 1999

$n=3$

Q How a no 0.457 can be displayed in 3½ digital meter in the ranges of (i) 0-1 volt (ii) 0-10 volt, (iii) 0-100 volt's.

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Range	Resolution	Sensitivity	Display
0-1 V	$1/10^3 = .001$	.001	0.457
0-10 V	$1/10^3 = .001$	.01	00.45
0-100 V	$1/10^3 = .001$	0.1	000.4

Error Analysis →

Types of Error

- Gross Error
  - Human Error
- Systematic Error
  - Enviromental
    - Temprature
    - Pressure
  - Instrumentational
    - Loading Effect
    - Defective instrument Effect
      - spring (flux is changing producing Error)
  - Reading (observational)
    - → Parallax Error
    - Zero adjustment or Max^m adjustment
- Random Error (Statistical)

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$$\boxed{\% \varepsilon_r = \frac{A_m - A_T}{A_T} \times 100}$$

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$$\% \text{ Error measured value} = \frac{\text{Full Scale Value} \times \% \text{ full scale error}}{\text{Measured Value}}$$

Q An ammeter has a full scale value of 100 amps and a full scale error of 2% than what is the error in the measurement of current of

(i) 50 amp.
(ii) 20 amp
(iii) 10 amp.

Ans (i) $\frac{100 \times 2}{50} \Rightarrow 4\%$.

(ii) $\frac{100 \times 2}{20} \Rightarrow 10\%$.

(iii) $\frac{100 \times 2}{10} \Rightarrow 20\%$.

The error increases at the lower value of the reading's if the error is spaified at full scale.

Composite Error →

$x_1 = a \pm \varepsilon_{r1}$

$x_2 = b \pm \varepsilon_{r2}$

$x_3 = c \pm \varepsilon_{r3}$

a) Sum or Difference Terms :–

$x = x_1 + x_2 + x_3$

$= x_1 + x_2 - x_3$

$$\% \varepsilon_r = \pm \left[\frac{a}{(a+b+c)} \varepsilon_{r1} + \frac{b}{(a+b+c)} \varepsilon_{r2} + \frac{c}{(a+b+c)} \varepsilon_{r3} \right]$$

(2) Product or division Terms →

$$x = \frac{x_1 x_2}{x_3}, \quad \frac{x_1}{x_2 x_3}$$

$$\boxed{\% \varepsilon x = \pm \left[\varepsilon_{r1} + \varepsilon_{r2} + \varepsilon_{r3} \right]}$$

(3) Power Terms →

$$x = x_1^m x_2^n x_3^p, \quad \frac{x_1^m x_2^n}{x_3^p}$$

$$\boxed{\% \varepsilon_r = \pm \left[m \varepsilon_{r1} + n \varepsilon_{r2} + p \varepsilon_{r3} \right]}$$

Uncertanity Error →

$$Y = f[x_1, x_2 \cdots x_n]$$

$w_{x1}, w_{x2}, \cdots w_{xn}$ are uncertanities of $x_1, x_2 \cdots x_n$.

Uncertanity of Y.

$$\boxed{w_y = \sqrt{\left(\frac{\partial y}{\partial x_1}\right)^2 w_{x_1}^2 + \left(\frac{\partial y}{\partial x_2}\right)^2 w_{x_2}^2 + \cdots}}$$

Standard deviation Error (σ)

$$Z = f(x_1, x_2 \cdots x_n]$$

$\sigma x_1, \sigma x_2 \cdots \sigma x_n$ are standard deviations of $x_1, x_2 \cdots x_n$.

Standard Deviation of Z.

$$\sigma_Z = \sqrt{\left(\frac{\partial Z}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial Z}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \cdots}$$

Q $R_1 = 10 \pm 2\%$. Three resistance's are
$R_2 = 20 \pm 3\%$. connected in series. find
$R_3 = 70 \pm 5\%$. Error in the measurement of
~~error~~ of total series resistance.

As
$$\%\epsilon_r = \frac{10}{100} \times 2 + \frac{20}{100} \times 3 + \frac{70}{100} \times 5$$
$$\Rightarrow \pm 4.3\%$$

$$R_{eq} = (10+20+70) \pm 4.3\%$$
$$\Rightarrow 100 \pm 4.3\%$$

Q Unknown resistance R is measured by using wheatstone bridge where $R = P/Q \cdot S$ the values of P, Q, S are $P = 10 \pm 2\%$, $Q = 20 \pm 3\%$, $S = 70 \pm 5\%$ find the error and limiting value.

As
$$\%\epsilon_r = [2+3+5] \Rightarrow 10\%$$
$$R \Rightarrow \frac{10 \times 70}{20} = \pm 10\%$$
$$\Rightarrow 35 \pm 10\%$$

Q The power consumed by the resistor of $(10 \pm 2)\%\ \Omega$ is measured by ~~using~~ passing a current through the resistance and the current is measured by using an ammeter that read's $(5 \pm 3)\%$ amps. find the power

and its limiting error? [250 ± 8%]

Ans

$$P = I^2 R$$

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$$\varepsilon_r = \pm [2\varepsilon_I + \varepsilon_R]$$

$$\Rightarrow \pm [2 \times 3 + 2]$$

$$\varepsilon_r = \pm 8\%$$

$$P = I^2 R \pm 8\% \Rightarrow \underline{250 \pm 8\%}\ W.$$

Q Find the uncertanity in the measurement of power dissipated by a resistor if the current flowing by resistor is 5 amp and the voltage across resistor is 200 volts. The uncertanity of ammeter is 0.2 amp and that of voltmeter uncertinity is 1.5 volt. Find the uncertanity of power.

Ans
$V = 200$, $I = 5A$.
$W_V = 1.5\ V$, $W_I = 0.2\ A$.

$$P = VI$$

$$\frac{\partial P}{\partial V} = I = 5$$

$$\frac{\partial P}{\partial I} = V = 200$$

$$W_P \Rightarrow \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 W_V + \left(\frac{\partial P}{\partial I}\right)^2 W_I^2}$$

$$\Rightarrow \sqrt{(5)^2 \times (1.5)^2 + (200)^2 \times (0.2)^2}$$

$$\rightarrow 40.5\ W$$

Accuracy → It is a closeness of instrument reading approaches to the true value of the quantity being measured

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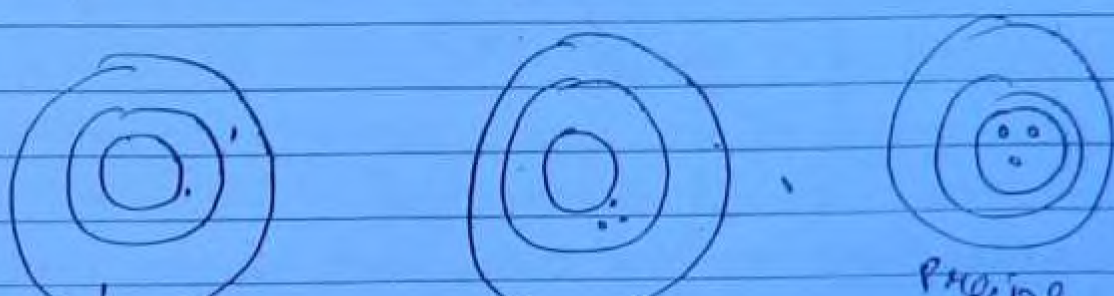
Precision → It is a measure of reproducibility and repeating of measurement. Precision is not guranteed for accuracy.
If the no of significant figure's ↑ses than the precision is higher

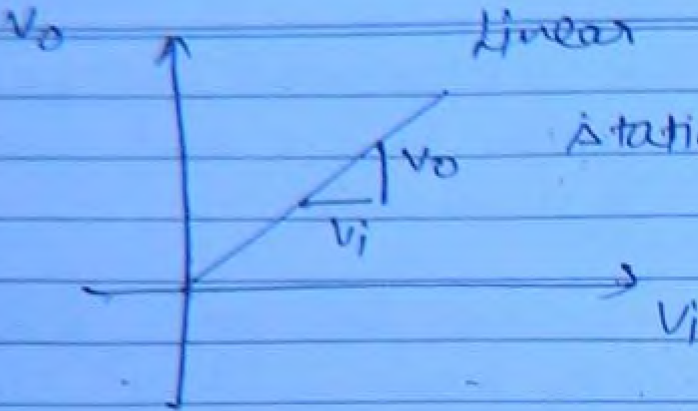


Not precise. Not accurate	precise Not Accurate	Precise Accurate

① 25 kV — 24 – 26 V – 1] — same precision
.025 kV — .024 – .026 V – 1
25.0 V — 24.9 – 25.1 – 0.1 → More precise.
25.00 V — 24.99 – 25.01 – .01 → Most precise

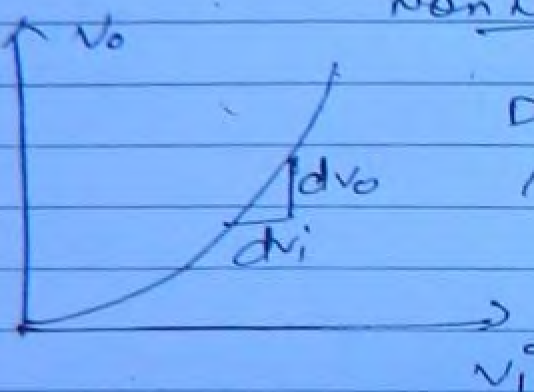
Linearity → If the output is proportional to input than it is called linear.
Sensitivity → It is the ratio of output to the input.
For linear devices the sensitivity is same value and hence it is called static sensitivity.

Linear



Static Sensitivity $= \frac{V_o}{V_i}$

Non Linear

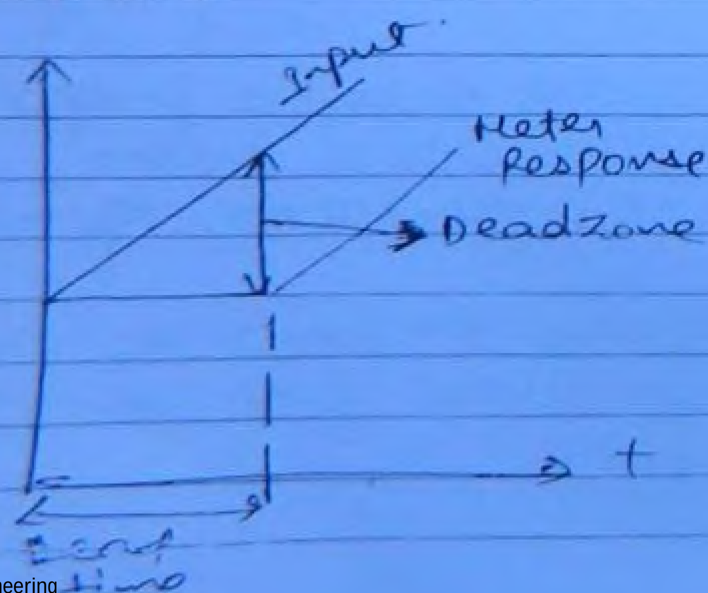


Dynamic sensitivity $= \frac{dV_o}{dV_i}$

Resolution → The smallest measurable qty of the input is called resolution

Dead Time & Dead Zone → The time req for the measurement to begin to respond to changes of the input quantity is called dead time.

Dead Zone is the largest change of i/p qty for which there is no output of instrument.



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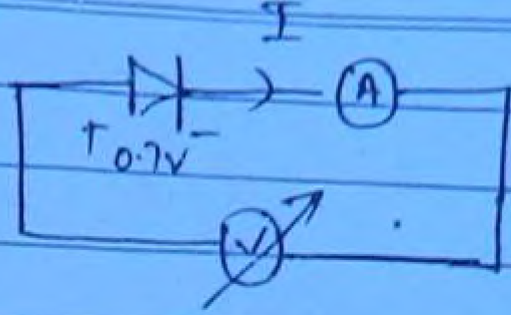
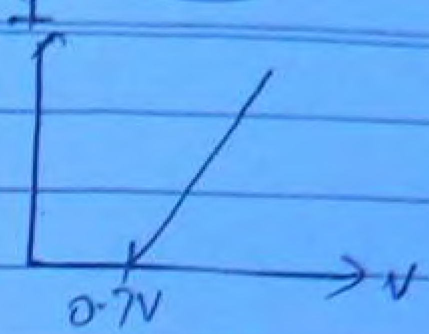




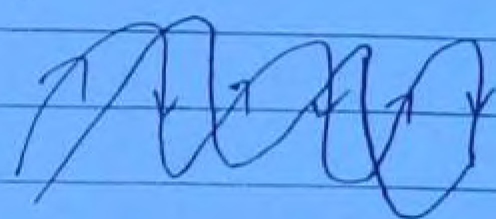
Properties of Resistance

(i) Resistance should not change with time.

(ii) The Resistivity should be higher and the temp^n coefficient α should be lower.

(iii) It is independent of frequency

(iv) Bifilar winding is used to eliminate the effect of inductance in the resistor.



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The End Book